



Senior High School Students' Metacognitive Awareness Process in Solving Absolute Value Problems

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ARTICLE INFO

ABSTRACT

Kata Kunci:

Metakognitif, Pemecahan, Masalah

Keywords:

Metacognitive, Solving, Problems

Proses kesadaran metakognitif terjadi dalam pembelajaran dan pemecahan masalah terhadap pemahaman. Situasi dan asumsi tentang masalah serta penilaian siswa menghubungkan pengetahuannya dengan apa yang dibutuhkan dalam masalah tersebut. Penelitian ini bertujuan untuk mendeskripsikan proses kesadaran metakognitif siswa SMA dalam menyelesaikan masalah nilai absolut. Pendekatan deskriptif dan eksploratif digunakan dalam penelitian kualitatif ini. Sebanyak 101 siswa MAN 3 Mandailing Natal diberikan soal nilai mutlak. Melalui tes, transkrip berpikir keras, dan wawancara, siswa dikategorikan berdasarkan kesadaran metakognitifnya. Hasilnya menunjukkan rendahnya kesadaran metakognitif dalam penggunaan diam-diam dan mengidentifikasi (26%). Proses kesadaran metakognitif sedang pada penggunaan semi sadar dan penggunaan sadar (69%). Sedangkan kesadaran metakognitif yang tinggi terdapat pada penggunaan strategis dan reflektif (5%). Dalam kesadaran metakognitif rendah, subjek memikirkan apa yang ditanyakan dan telah memikirkan informasi dalam pertanyaan. Pada kesadaran metakognitif sedang, subjek telah mencapai tahap mengaitkan informasi untuk memecahkan masalah nilai absolut dan memikirkan kembali fase atau langkah (sudah mempunyai strategi) dalam menyelesaikan masalah tersebut. Dalam kesadaran metakognitif tinggi, subjek telah memikirkan berbagai teknik, memiliki proses yang unik, dan sampai pada tahap memikirkan kembali uraian jawaban atas permasalahan yang telah dipecahkan.

The process of metacognitive awareness occurs in learning and problem-solving against understanding. Situations and assumptions about the problem and students' judgments relate their knowledge to what is needed in the problem. This study aimed to describe the metacognitive awareness process of senior high school students solving absolute value problems. An exploratory, descriptive approach is involved in this qualitative research. As many as 101 MAN 3 Mandailing Natal students were given the absolute value problem. Through tests, think-aloud transcripts, and interviews, students are categorized based on their metacognitive awareness. The results showed low metacognitive awareness in tacit and identify use (26%). The process of medium metacognitive awareness is in semi-aware use and aware use (69%). While high metacognitive awareness is on strategic and reflective use (5%). In low metacognitive awareness, the subject thinks about what is being asked and has thought about the information in the question. In medium metacognitive awareness, the subject has reached the stage of relating the information to solve the absolute value problem and rethinking the phase or step (already have strategies) in solving the problem. In high metacognitive awareness, the subject has thought of

various techniques, has a unique process, and to the stage of rethinking the description of the answers to the problems that have been solved.

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INTRODUCTION

Metacognitive is a term introduced by Flavell. Flavell defines metacognitive as knowledge about cognitive objects, namely everything related to cognition.(Flavell, 1979) Metacognitive refers to a person's ability to know and manage cognitive processes.(Schraw & Moshman, 1995) Kelemen et al. state that metacognitive refers to knowing and monitoring the learning process.(Kelemen et al., 2000) Then, Gourgey emphasized that metacognitive is knowledge and understanding of how one learns, awareness when one understands and is not understood, knowledge of how to use available information to achieve goals, the ability to assess cognitive needs in various exercises, knowledge of strategies used to achieve goals, measure a person's progress either during or after it is carried out.(Kılınç, 2013) Thus, metacognitive is a person's knowledge and understanding of his thought processes, the ability to monitor and direct (regulate) the processes and results of his thinking, and to evaluate his thinking processes and outcomes.

Metacognitive is a crucial part of students' learning activities and solving problems. It was reinforced by Schraw's statement (Schraw, 1998), which confirms that metacognitive is essential for successful learning because it allows individuals to manage their cognitive skills and determine their weaknesses which can be corrected by building new cognitive skills. During the learning process, metacognitive skills enable students to select appropriate strategic interventions, monitor the implementation of strategies, and evaluate their effectiveness.(Cao & Nietfeld, 2007) Aurah et al. also stated that metacognitive is essential in problem-solving because it includes knowledge about problem-thinking, monitoring, and regulating cognitive processes.(Aurah et al., 2011) O'Neil & Brown stated that metacognitive is a process in which a person thinks about thinking to build strategies to solve problems. If someone has good metacognitive skills, he will be successful in learning.(O'Neil Jr. & Brown, 1998)

Metacognition is indeed a fundamental aspect of students' learning and problem-solving abilities. Underscores its importance by highlighting how metacognitive skills enable individuals to not only manage their cognitive resources effectively but also identify and rectify any cognitive weaknesses through the development of new skills. This reflective process is crucial in educational settings as it empowers learners to take control of their learning experiences. Emphasize that metacognitive skills play a pivotal role during the learning process by facilitating students' ability to choose appropriate strategies for solving problems. This selection process involves a conscious evaluation of available strategies and their potential effectiveness in achieving desired outcomes. Moreover, metacognitive skills enable learners to monitor the implementation of chosen strategies, allowing for adjustments as necessary to optimize learning and problem-solving outcomes.

further elaborate on the significance of metacognition in problem-solving contexts. They argue that metacognitive awareness includes not only knowledge about problem-solving strategies but also the ability to monitor and regulate cognitive processes throughout the problem-solving journey. This self-regulatory aspect ensures that learners can adapt their approaches dynamically in response to evolving challenges or insights gained during the problem-solving process. Expanding on this, metacognition as a cognitive process wherein individuals actively engage in "thinking about thinking" to develop effective strategies for solving problems. They posit that strong metacognitive skills contribute significantly to

academic success by fostering a deeper understanding of how to approach complex tasks systematically and strategically. By engaging in metacognitive reflection, learners can refine their problem-solving methodologies and enhance their overall learning experiences. In practical terms, fostering metacognitive development among students involves creating environments that encourage reflection, self-assessment, and goal-setting. Educators can support this process by integrating activities that prompt students to articulate their thinking processes, evaluate their problem-solving strategies, and consider alternative approaches. By doing so, learners not only become more adept at navigating academic challenges but also develop lifelong skills that are transferable across various domains of learning and problem-solving. In conclusion, the integration of metacognitive practices into educational contexts is crucial for nurturing students' abilities to manage their cognitive resources effectively, select appropriate strategies, and monitor their progress towards achieving learning objectives. By promoting metacognitive awareness and skills, educators empower learners to become proactive agents in their own learning journeys, equipped with the tools necessary to tackle complex problems and succeed in diverse academic and professional pursuits.

Schraw and Dennison stated that metacognitive awareness allows individuals to plan, sequence, and monitor their learning in ways that directly enhance performance. (ÇETİN et al., 2012) Students who have metacognitive awareness are more strategic and perform better than students who are not metacognitive aware. That was supported by Abdellah, who states that there is a positive relationship between metacognitive awareness and academic achievement. (Abdellah, 2015) Metacognitive awareness affects students' reasoning in solving problems, so students will be more careful in solving problems. Ramirez et al. argue that students' metacognitive awareness is embedded in students who can solve various problems. (Ramirez-Corona et al., 2013) Based on the above, it shows that metacognitive awareness is needed by someone to solve problems. Thus students with high metacognitive awareness will be better than students with low metacognitive awareness in solving problems. Magiera & Zawojewski and Wilson & Clarke arrange metacognitive awareness activities related to (1) thinking about what is known (tasks, special knowledge, relevant mathematical knowledge, or strategies in solving problems), (2) thinking about the position of the problem in the problem-solving process, (3) thinking about what else needs to be done, (4) thinking about what can be done. (Magiera & Zawojewski, 2011) (Wilson, 2019) The student's metacognitive awareness can be seen when carrying out the process of solving mathematical problems.

Identifying students' metacognitive awareness should be based on how students solve problems. This study focuses on the process of metacognitive awareness of students in absolute value. Many math teacher candidates still have difficulty solving absolute value problems. Students' difficulties in solving absolute value problems allow for differences in students' metacognitive awareness in solving absolute value problems. Adinda et al. described six levels of student metacognitive awareness in problem-solving. However, the process of metacognitive awareness in solving problems based on that level has not been detailed sustainably. Therefore this study will describe the metacognitive processes of mathematics teachers in solving absolute value problems based on Adinda et al.'s metacognitive awareness processes. (Adinda, 2022)

Identifying students' metacognitive awareness in problem-solving, particularly in contexts involving absolute value, is essential for understanding how learners approach and navigate mathematical challenges. The focus of this study is to explore the intricacies of metacognitive processes among mathematics teacher candidates when solving absolute value problems, a topic known to present difficulties for many learners. The complexity of absolute value problems often stems from the need to consider both positive and negative solutions, requiring a nuanced understanding and application of algebraic principles. Categorized metacognitive awareness in problem-solving into six distinct levels, offering a framework to assess the depth and effectiveness of students' problem-solving strategies. However, while

these levels provide a theoretical foundation, practical insights into how these processes manifest in the specific context of absolute value problems remain underexplored. This study aims to elucidate how mathematics teacher candidates employ metacognitive strategies to tackle absolute value problems. This includes exploring how candidates initiate problem-solving tasks, monitor their progress, and adjust their approaches based on feedback and insights gained during the process. Understanding these cognitive processes is crucial for educators as it can inform instructional practices aimed at enhancing students' metacognitive competencies.

The study will employ qualitative research methods, such as interviews and observations, to capture the rich nuances of metacognitive awareness in action. Participants will be asked to verbalize their thought processes as they work through a series of absolute value problems, providing researchers with valuable insights into the decision-making strategies employed at each stage of problem-solving. Furthermore, the research will investigate any challenges or misconceptions that mathematics teacher candidates encounter when applying metacognitive strategies to absolute value problems. These insights can highlight areas where additional support or instructional interventions may be beneficial to enhance students' metacognitive development and improve their problem-solving efficacy. In conclusion, by examining how mathematics teacher candidates navigate absolute value problems through the lens of metacognitive awareness, this study aims to contribute to both theoretical understanding and practical applications in mathematics education. The findings have the potential to inform curriculum design, instructional strategies, and teacher training programs focused on fostering robust metacognitive skills among future educators and their students alike.

METHOD

This qualitative research aims to explore the process of metacognitive awareness of mathematics teacher candidates through students' writing, thinking, and speech while solving absolute value problems. (Mockford, 2008) This research involved 101 semester VI students of Malang's Mathematics Education Study Program. The selected students are semester VI students because, in semester VI, students have studied the problem of absolute value. The absolute value problem selected in this study is a problem related to the set of solutions in absolute value equations or inequalities, as presented in Figure 1. Furthermore, the data obtained is described in depth and systematically to accurately describe students' metacognitive awareness in solving value problems. Absolute. The study involves 101 semester VI students specifically chosen because this academic phase typically marks their exposure to and familiarity with absolute value problems within their curriculum. This timing ensures that participants have sufficient background knowledge and experience to engage meaningfully with the selected absolute value problems presented in Figure 1. These problems are carefully selected to encompass a range of scenarios involving absolute value equations or inequalities, thus providing a comprehensive basis for evaluating students' metacognitive awareness.

Data collection in this study focuses on capturing students' metacognitive processes through multiple channels: their written responses, verbal explanations during problem-solving sessions, and reflective thoughts articulated during interviews or focus group discussions. These diverse sources of data enable researchers to triangulate findings and gain a holistic understanding of how students perceive, approach, and navigate absolute value problems. Moreover, the qualitative nature of the research allows for in-depth exploration and systematic analysis of the collected data. Each participant's responses and behaviors are meticulously examined to identify recurring patterns, unique strategies, challenges encountered, and instances of effective metacognitive regulation. This systematic approach

ensures that the complexities of students' metacognitive awareness in relation to absolute value problem-solving are accurately documented and comprehensively understood.

Furthermore, the findings from this research endeavor will be presented and interpreted with a focus on providing nuanced insights into the cognitive and metacognitive processes at play. By shedding light on the strategies employed by mathematics teacher candidates, educators can gain valuable insights into instructional approaches that effectively foster metacognitive development in future teachers. In conclusion, this study aims to contribute significantly to the field of mathematics education by illuminating how mathematics teacher candidates navigate and understand absolute value problems through the lens of metacognitive awareness. By uncovering the intricacies of students' problem-solving approaches and metacognitive reflections, the research seeks to inform educational practices aimed at enhancing both teaching and learning experiences in mathematics classrooms.

Data analysis was in the form of students' written assignments in solving absolute value problems based on the correctness of the completion carried out by the subject guided by the complete instructions and the key. The subject's answers were analyzed based on predetermined indicators. Subjects were assigned a particular metacognitive awareness based on their answers and records of students' activities on think load in solving problems and then clarified by interviews.

$$\text{Find all the values of } x \text{ that satisfy } |x - 1| + x^2 - 3x \geq -\frac{13}{4}$$

FIGURE 1. Absolute Value Problem

RESULT AND DISCUSSION

Based on the completion activities of students on absolute value problems in think-aloud and interviews, metacognitive awareness is classified into low metacognitive awareness, medium metacognitive awareness, and high metacognitive awareness. Students categorized with low metacognitive awareness typically demonstrate limited engagement in metacognitive processes during problem-solving. They may struggle to articulate their thought processes coherently or fail to monitor their own understanding effectively. For instance, these students might approach absolute value problems mechanically, applying rote methods without considering alternative strategies or reflecting on the effectiveness of their approaches. Their responses in think-aloud sessions and interviews may reveal a reliance on surface-level strategies or a lack of self-regulation in adjusting their problem-solving tactics based on feedback or new information.

Medium Metacognitive Awareness: Students classified with medium metacognitive awareness exhibit a moderate level of engagement in metacognitive processes. They demonstrate some ability to monitor their progress and apply strategies purposefully when solving absolute value problems. These students may articulate their approaches more clearly during think-aloud sessions, showing awareness of different problem-solving techniques and occasionally reflecting on the reasoning behind their choices. While they may not consistently employ advanced metacognitive strategies, they show potential for growth by adjusting their strategies based on task demands and demonstrating some self-awareness of their problem-solving behaviors. **High Metacognitive Awareness:** Students identified with high metacognitive awareness demonstrate robust engagement in metacognitive processes throughout their problem-solving activities. They exhibit a deep understanding of various problem-solving strategies related to absolute value problems and consistently monitor their thinking processes. These students are adept at articulating their reasoning during think-aloud sessions, providing detailed explanations of their strategic choices and demonstrating flexibility in adapting their approaches as needed. They exhibit strong self-regulation skills,

actively evaluating the effectiveness of their strategies and making adjustments to optimize their problem-solving outcomes.

By systematically classifying students' metacognitive awareness into these categories, researchers can identify patterns and trends in how individuals approach and navigate absolute value problems. This classification system not only helps in understanding the cognitive development of mathematics teacher candidates but also informs educators about effective strategies for fostering metacognitive skills in diverse learning environments. Furthermore, the findings from these classifications can guide targeted interventions and instructional approaches aimed at enhancing students' metacognitive awareness. Educators can design learning activities that scaffold students' metacognitive development, providing opportunities for practice, reflection, and feedback to support progression from lower to higher levels of metacognitive proficiency.

In summary, the categorization of metacognitive awareness levels among mathematics teacher candidates provides valuable insights into their problem-solving behaviors and cognitive strategies when confronted with absolute value problems. This research contributes to the broader goal of improving mathematics education by highlighting the importance of metacognitive skills in fostering deeper understanding and proficiency in mathematical problem-solving. The process of metacognitive awareness of students is categorized into Tacit and Identify Use, Semiaware and Aware Use, and Strategic and Reflective Use. A more detailed classification is presented in Table 1 below.

TABLE 1. Classification of metacognitive awareness in absolute values

Classification	Category	Number of participants	Participant code
Low Metacognitive Awareness	Tacit and Identify Use	26	S12
Medium Metacognitive Awareness	Semiaware and Aware Use	70	S24
High Metacognitive Awareness	Strategic and Reflective Use	5	S26

Based on table 1 shows that the process of low metacognitive awareness is in tacit use and identify use (26%). The process of medium metacognitive awareness is in semi-aware use and aware use (69%). While high metacognitive awareness is on strategic and reflective use (5%). The process of metacognitive awareness of students in solving absolute value problems can be explained as follows.

Low Metacognitive Awareness (LMA)

S12's written answer shows that S12 solves the problem using only one condition, $x \geq 1$. In problem 1, S12 only applies one possibility. There should be one more possible inequality. S12 finds the roots of the inequality using the ABC formula. S12 doesn't rethink the new inequality it has created. During troubleshooting, S12 did think aloud, and the process was transcribed. Some parts of the think-aloud S12 transcript are as follows.

Translated version:

First of all, I read the problem and understand it. What you are looking for is the value of x . Since there is an absolute $x-1$, make it for $x \geq 1$. Because it looks easy. I tried it first.. but.. after I tried it turned out that there were a few problems because the equation couldn't be factored in the usual way. So you have to use the abc formula. Just got the results.

FIGURE 2. LMA's Think aloud

Based on the think-aloud transcript, the underlined sentences show that S12 re-read the questions and tried to rethink what the questions meant. S12 draws information from the absolute value sign and creates possible inequalities or equations from the absolute value sign. S12 said that the questions looked easy. That shows that S12 already knows how to find solutions to inequalities. But S12 cannot draw other information from the new form of inequality, so he looks for the roots of the inequality using the ABC formula. That shows that S12 already has information but doesn't think about how to use the knowledge he gets to find a solution. Based on this, it seems that S12 has thought of the information, but S12 did not use all the information he obtained to solve the problem. Next, S12 was interviewed to get confirmation of the written answers and think-aloud he made.

- R : *Do you understand what is being asked?*
S12 : *Understood, ma'am.*
R : *Try to state what is asked in the problem!*
S12 : *Everyone is looking for the value of x from the equation.*
R : *How do you know what to ask?*
S12 : *From the question ma'am*
R : *What information do you know from the questions?*
S12 : *This problem has an absolute value and quadratic inequality.*
R : *What did you get from this information?*
S12 : *I know that two possibilities can be made if there is an unmistakable sign.*
R : *Yes. How do you find out this information?*
S12 : *From the form of the question, ma'am.*
R : *From the answer you made, you only made one possibility.*
S12 : *Yes, ma'am. I only take one possibility.*
R : *Why?*
S12 : *I think one possibility is already possible, ma'am... Do you have to do both, ma'am?*

It was in line with the opinion of Çiltaş & Tatar, which states that students solve problems as if there were no absolute values in the given equations and inequalities.(ÇILTAŞ, 2020) Permata & Prabawanto also stated that students solve absolute value problems by making absolute signs less common and operating them like ordinary brackets.(PERMATA & PRABAWANTO, n.d.) It also shows that students do not understand absolute sign operations. Adinda et al. stated that students failed to solve absolute value problems because students were unable to draw complete information.(Adinda et al., 2021) Sa'dijah et al. explained that students' low problem-solving ability is related to low numeracy skills.(Adinda et al., 2022)

Medium Metacognitive Awareness (MMA)

S24 solves the problem by dividing the absolute sign into two cases. Of the two cases, S24 has shown that there is information obtained. Then S24 connects the information he gets by describing each case so that he gets the value x . S24 uses the absolute value definition and properties to describe the absolute value sign. The S24 further simplifies each case. S24 can find new information obtained from new inequalities for each case. S24 finds that every inequality can be satisfied for all real numbers x . So S24 concludes that all $x \in \mathbb{R}$ satisfy the inequality. During troubleshooting, S24 did think aloud, and the process was transcribed. The transcript of think-aloud S24 is as follows.

Translated version:

For the first problem, find all values of x that satisfy $|x - 1| + x^2 - 3x \geq -\frac{13}{4}$. I do that by shifting terms that are not absolute values. So the absolute price is on the left $|x - 1| \geq -\frac{13}{4} + 3x - x^2$. Then to remove the absolute value, so $x - 1 \geq -\frac{13}{4} + 3x - x^2$ or $-(\frac{13}{4} + 3x - x^2) \geq x - 1$. Then it is done, processed so that the two equations are greater than 0. The first is $\frac{9}{4} - 2x + x^2 \geq 0$ and the second is $\frac{17}{4} - 4x + x^2 \geq 0$. For the first equation, $\frac{9}{4} - 2x + x^2 \geq 0$ to check if it is greater than zero, I think it is $x^2 - 2x \geq 0$ if x is negative it will comply. Because negative squared will be positive. And -2 times a negative will also always be a positive value. So if you add 9/4, the negative x will satisfy. Then if x is 0 it will also fulfill because its value becomes 9/4. Which produces a negative value if $x < 2$ and more than zero. Now for $-2x + x^2$, I tried to find the first derivative, found the minimum value, and found $x=1$. Plugging in $x=1$, the result is $-2x + x^2 = -1$, but -1 is still positive when you add 9/4. So fulfill. So $x > 2$ will also be positive because x^2 is bigger, so adding 9/4 is also positive. So satisfies for all real numbers. So the second one is $\frac{17}{4} - 4x + x^2 \geq 0$, checking is also the same as before. So number 1, the x value that satisfies that is all $x \in \mathbb{R}$. For number one, I checked because the two inequalities cannot be factored, if I look for the roots using the abc formula, I am confused as to whether the inequality is greater than zero or not. So I use the manual one starting from negative x , zero and positive.

FIGURE 3. MMA's Think aloud

Based on think-aloud, S24 first read the questions carefully. S24 simplifies the inequality so that the absolute value sign is on the left. It shows that S24 is trying to obtain new information from its obtained information. S24 also knows what to do regarding the information he has received. S24 connects information with prior knowledge to find a way to find a solution by describing each case. That shows that S24 has thought of the information and can relate to it to solve the problem. S24 analyzes each term of the inequalities individually so that S24 can conclude that each inequality can always be satisfied for all $x \in \mathbb{R}$. That means that S24 thinks about the strategic moves it makes. Next, S24 was interviewed to get clarification on the answers he made. The following is an excerpt from an interview with S24 based on solving the problem.

- R : *What information do you know about the problem?*
S24 : *If you look at the problem, the form of the equation or the inequality has an absolute value. And what to look for.*
R : *Is the information you get helpful in solving the problem?*
S24 : *very useful, because if we don't know the similarities or we don't know what to look for, of course, we will experience difficulties and confusion in solving problems.*
R : *How do you connect the information you get to solve the problem?*
S24 : *the information obtained, such as the given equation, determines the steps to solve the problem.*
R : *how do you solve the problem?*
S24 : *so to solve this problem, because the equations and inequalities contain all absolute values, so first of all, remove or describe the absolute values first, then after obtaining equations or inequalities that do not contain absolute values then, look for a solution or x value that satisfies,*
R : *Does your problem-solving strategy follow similar problem-solving steps?*
S24 : *I think it is appropriate.*
R : *what are the steps you made? Are they correct?*
S24 : *In general, the steps that I take are like this, but as for the accuracy of the answer, I can't be sure because there may be errors in the process or the calculations.*

The intermediate ability of students is related to students' prior knowledge.(Sa'dijah et al., 2023) That was evident from their initial understanding of this absolute material, both

conceptual and procedural, in the problem-solving process. Furthermore, Adinda et al. explained that students with intermediate abilities tend to be in the semi-aware and aware use categories. (Hidayanto & Rahmatina, 2020) In this intermediate ability, students tend to have passed the understanding phase in solving problems. (Adinda et al., 2023)

High Metacognitive Awareness (HMA)

S26 solves the problem using the absolute value definition. S26 creates two cases by definition. In each case, S26 describes the inequality to obtain a simpler one. Based on the new inequality in each case, S26 finds that the left side of the inequality will always be larger than the right. Therefore, S26 concludes that $x \in \mathbb{R}$ holds for inequalities. Furthermore, S26 also made another strategy to solve the problem. S26 translates the inequality into another form so that from the new inequality, it is clear that the left side is always more significant than the right side for all $x \in \mathbb{R}$. That was also confirmed based on think-aloud; S26 said again what to look for in the problem. S26 tries to find the value of x that satisfies the inequality. It shows that the S26 has thought about what was asked.

Furthermore, in problem 1, S26 uses two cases to find solutions to inequality. S26 describes each case and simplifies inequalities that reveal solutions. In a second way, S26 changes the initial form of the inequality. So from the modified form, it can show the solution to the inequality. It can be observed in the following HMA think-aloud transcript.

Translated version:
 From this problem, find x that satisfies $|x - 1| + x^2 - 3x \geq -\frac{13}{4}$. Initially, we divided the cases, namely the conditions from the absolute first. $|x-1|$ then the case is that the first $x-1 \geq 0$ and the second $x-1 < 0$. Because $x-1 \geq 0$ then $x \geq 1$, so the initial inequality $|x - 1| + x^2 - 3x \geq -\frac{13}{4}$ turns into $x - 1 + x^2 - 3x \geq -\frac{13}{4}$. Then we make one segment $x^2 - 2x - \frac{4}{4} + \frac{13}{4} \geq 0$. So the inequality turns out to be $x^2 - 2x + \frac{9}{4} \geq 0$. Now, this inequality turns into a quadratic inequality... I'll try to... I think this can be changed to $(x - 1)^2 + \frac{5}{4} \geq 0$. Now for this, because earlier because we already have $x \geq 1$ and for this one, whatever the value of x is, it must be positive, so this inequality is $x \geq 1$... let's try to draw it.
 Then for the second case, namely $x-1 < 0$ becomes $x < 1$ then this can be .. the initial inequality changes to $-x + 1 + x^2 - 3x \geq -\frac{13}{4}$ changes to .. move sides $x^2 - 4x + \frac{17}{4} \geq 0$. The inequality can turn into a perfect square $(x - 2)^2 + \frac{1}{4} \geq 0$ Jadi $(x - 2)^2 \geq -\frac{1}{4}$ in my opinion, whatever the value of x occurs. So from this second case the condition is $x < 1$. From cases 1 and 2, they can be combined so that $x \geq 1$ and $x < 1$ are combined to become x from $-\infty$ to ∞ .
 Let's try another method without working on cases 1 and 2. The problem can be translated into another form, namely $|x - 1| +$ perfect square of $x^2 - 3x$ So $(x - \frac{3}{2})^2 - \frac{9}{4} \geq -\frac{13}{4}$. $-\frac{9}{4}$ move segments. So the form of the problem from the start can change to $|x - 1| + (x - \frac{3}{2})^2$ which is a perfect square ≥ -1 . Now whatever the value of x is in absolute value and in the square it will always be positive and greater than -1 .

FIGURE 4. HMA's Think-aloud

Based on think-aloud, S26 uses the first way by definition, namely looking at the possibility of x from two cases. In a second way, he changes the initial form of the inequality by using perfect squares. From this perfect square, S26 finds a more straightforward form of inequality that can show the solution to the inequality. S26 has also taken steps to resolve it by following the previous problem and examining the solutions it obtained. It means S26 rethinks its strategic steps, thinks of different ways of solving problems, and rethinks the correctness of the mathematical answers that have been solved. Next, S26 was interviewed to get confirmation of his written responses and think aloud. The following is an excerpt from an interview with S26 based on problem-solving.

R : What information do you know about the problem?

S26 : Questions in the form of absolute value inequalities. Inequality contains absolute values and quadratic inequalities.

- R : *Is the information you get helpful in solving the problem?*
S26 : *Yes, it is handy because, from this information, we can determine what steps we will use to find the value of x*
- R : *How do you connect the information you get to solve the problem?*
S26 : *Based on the absolute value information, I use the absolute value definition, wherein the absolute value of an x , I can make two cases to find the probability of x . From this definition, I connect with the problem at hand. Then if there is a quadratic form, I look for the simple quadratic form by squaring.*
- R : *How do you solve the problem?*
S26 : *I use two methods; the first is definitions, namely looking at the probability of x from two cases. After I described each case, I found the x value of the inequality. Combined with both cases, I found a solution. The second way I change the initial form of the inequality is by using perfect squares. From there, I found a more straightforward form of inequality that can show the solution to inequality.*
- R : *Does your problem-solving strategy follow similar problem-solving steps?*
S26 : *Yes, it is appropriate.*
- R : *What are the steps that you made? Were they correct?*
S26 : *Yes*
- R : *Why did you choose this strategy?*
S26 : *Following the method that I have learned*
- R : *Have you tried other strategies before?*
S26 : *I use two ways*
- R : *Are you sure about your answer?*
S26 : *Sure. I've checked the value of x , which I found is following the problem.*

High-ability students can solve problems differently, distinguish the process of obtaining accurate information, and think of effective problem-solving strategies. (Adinda et al., 2023) It follows research by Adinda et al & Purnomo et al. that high ability can modify strategies and reasons in detail. (Purnomo et al., 2023) Rahmatina et al. further emphasized that students with high abilities can routinely apply prior knowledge to the contextual problems they face in decision-making. (DESI et al., 2022)

CONCLUSION

Based on the results of this study shows that the process of low metacognitive awareness is in tacit use and identify use (26%). The process of medium metacognitive awareness is in semi-aware use and aware use (69%). While the process of high metacognitive awareness is on strategic use and reflective use (5%). In low metacognitive awareness, the subject thinks about what is being asked and has thought about the information in the question. In medium metacognitive awareness, the subject has reached the stage of thinking about relating the information to solving the absolute value problem and rethinking the stage or step (already have strategies) in solving the problem. In high metacognitive awareness, the subject has thought of various strategies, has a unique strategy, and is rethinking the description of the answers to the problems that have been solved.

This research implies that in solving the problem, students value is in classifying metacognitive awareness into different categories. Follow-up in tertiary institutions in the selection of absolute value material learning models can be considered based on the classification and categories of the student's metacognitive awareness so that it has an impact on the metacognitive abilities of senior high school students. In tertiary education settings, the findings from this research can influence the design and selection of learning models and materials related to absolute value topics. Tailoring instructional approaches to align with students' metacognitive awareness levels can effectively enhance their metacognitive abilities

and overall problem-solving skills. For instance, students identified with low metacognitive awareness may benefit from structured activities that explicitly teach and scaffold metacognitive strategies, such as think-aloud protocols, self-assessment tools, and guided reflections on problem-solving processes. These interventions can help these students develop a deeper understanding of when and how to apply various problem-solving techniques related to absolute value. On the other hand, students classified with medium metacognitive awareness can benefit from activities that encourage them to reflect more deeply on their problem-solving strategies and consider alternative approaches. Educators can foster their metacognitive growth by providing opportunities for peer discussions, where students can share and evaluate different problem-solving methods, thus broadening their repertoire of strategies and enhancing their ability to self-regulate.

For students with high metacognitive awareness, advanced learning models and materials can challenge them to delve deeper into the theoretical underpinnings of absolute value concepts. These students may thrive in environments that emphasize critical thinking, creative problem-solving, and the application of abstract mathematical principles to real-world scenarios. By catering to their advanced metacognitive abilities, educators can nurture their potential as future mathematics educators who can model effective problem-solving strategies and support their own students' metacognitive development. Additionally, integrating metacognitive awareness classifications into curriculum planning and instructional strategies can create a more inclusive and differentiated learning environment. It allows educators to address the diverse needs and learning preferences of students effectively, fostering an equitable learning experience where all students have the opportunity to achieve academic success. In conclusion, leveraging the insights gained from categorizing students' metacognitive awareness in solving absolute value problems can significantly impact educational practices at the tertiary level. By aligning learning models and materials with students' metacognitive capabilities, educators can promote deeper engagement, enhance problem-solving skills, and ultimately empower students to become proficient mathematics learners and educators in their own right. This approach not only enhances academic achievement but also cultivates lifelong learning habits that extend beyond the classroom.

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