EXPLORING MALAYSIAN STUDENTS' MATHEMATICAL THINKING SKILLS

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Abstract

Mathematical thinking is a fundamental aspect of mathematics education and encompasses a range of cognitive processes. It is a crucial part of mathematics for students who desire to continue higher level mathematics. This research aims to investigate the students' mathematical thinking based on the four elements of mathematical thinking, namely specializing, generalising, conjecturing and convincing. In addition, this study also aims to determine which element of mathematical thinking is the most difficult among students. This study employed a nonexperimental descriptive research design, and fifty (50) Form Four students from a public school in Johor, Malaysia, have been chosen as the sample of this study. A Mathematical Thinking Test (MTT) was used as an instrument in this study to collect the data. The test results showed that students are lacking in mathematical thinking skills, especially the conjecturing element, which was the most difficult element among the students. Therefore, teachers and other relevant authorities should pay closer attention to this issue. This study suggests that it is necessary to find solutions to overcome student difficulties in learning mathematics. Besides, this study established the instrument's validity and reliability in assessing mathematical thinking. Hence, it can be replicable and practical to be used to measure secondary school students' mathematical thinking.

***Keywords**: Mathematics Education; Mathematical Thinking; Mathematical Test Instrument; Rasch Model, Thinking Ability*

Abstrak

Pemikiran matematis adalah aspek mendasar dari pendidikan matematika dan mencakup serangkaian proses kognitif. Ini adalah bagian penting dari matematika bagi siswa yang ingin melanjutkan matematika tingkat yang lebih tinggi. Penelitian ini bertujuan untuk menyelidiki pemikiran matematis siswa berdasarkan empat unsur berpikir matematis, yaitu mengkhususkan, menggeneralisasi, menduga, dan meyakinkan. Selain itu, penelitian ini juga bertujuan untuk mengetahui unsur berpikir matematis mana yang paling sulit dimiliki siswa. Penelitian ini menggunakan desain penelitian deskriptif non-eksperimental, dan lima puluh (50) siswa Kelas Empat dari sebuah sekolah negeri di Johor, Malaysia, dipilih sebagai sampel penelitian ini. Tes Berpikir Matematis (MTT) digunakan sebagai instrumen dalam penelitian ini untuk mengumpulkan data. Hasil tes menunjukkan bahwa kemampuan berpikir matematis siswa kurang terutama pada unsur dugaan yang merupakan unsur tersulit di kalangan siswa. Oleh karena itu, guru dan otoritas terkait lainnya harus memberikan perhatian lebih terhadap masalah ini. Penelitian ini menyarankan perlunya mencari solusi untuk mengatasi kesulitan siswa dalam belajar matematika. Selain itu, penelitian ini menetapkan validitas dan reliabilitas instrumen dalam menilai berpikir matematis. Oleh karena itu, dapat ditiru dan praktis untuk digunakan untuk mengukur pemikiran matematis siswa sekolah menengah.

***Kata kunci**: Pendidikan Matematika; Berpikir Matematis; Instrumen Tes Matematika; Model Rasch; Kemampuan Berpikir.*

INTRODUCTION

The objective of mathematics education in school extends beyond rote learning and computational skills. It aims to foster students' mathematical thinking skills, enabling them to reason, problem-solve, and apply mathematics in real-life contexts (Ilyas et al., 2022, Calder et al., 2021; Bakker et al., 2021). Mathematical thinking encompasses a range of cognitive processes involved in problem-solving reasoning, and conceptual understanding. Over the past decade, numerous studies have explored the definition of mathematical thinking, and the concept of thinking mathematically, and proposed various theories to explain this cognitive phenomenon (Celik & Ozdemir, 2020). Mathematical thinking is an important aspect of cognitive development, and it involves four interconnected stages: concrete, semi-concrete, semi-abstract, and abstract (Khalil, 2019). However, the study discovered a substantial link between critical thinking and higher-order thinking, reasoning, mathematical thinking skills, and problem-solving sub-dimensions (Celik & Ozdemir, 2020).

One prominent definition of mathematical thinking comes from Schoenfeld (2016), who describes it as "the cognitive process of analysing, abstracting, and generalizing patterns and structures in mathematical situations." This definition emphasizes the active engagement of individuals in making connections, recognizing patterns, and developing generalizations, all of which are essential components of mathematical thinking. Schoenfeld's definition highlights the importance of higher-order cognitive skills and the ability to transfer mathematical knowledge to different contexts. According to Schoenfeld (2016), learning to think mathematically involves developing a mathematical point of view, valuing the processes of mathematization and abstraction, and having the ability to make sense of mathematical concepts. Mathematical thinking involves creating and testing conjectures, making sense of objects, and forming arguments (Mukuka, Balimuttajjo & Mutarutinya, 2023). Mathematical thinking is important for developing critical thinking skills, reasoning and proof, creating and testing conjectures, making sense of mathematical concepts, and cognitive development. The importance of mathematical thinking has been argued by previous researchers in various ways, for instance, Celik and Ozdemir (2020) suggest that mathematical thinking is a predictor of critical thinking dispositions of preservice mathematics teachers.

The importance of mathematical thinking skills in promoting higher-order thinking and metacognition has been highlighted in recent studies. In their research on mathematical thinking and metacognition, Swidan and Tall (2019) argued that mathematical thinking fosters cognitive processes such as reflection, monitoring, and self-regulation. They found that students who engage in mathematical thinking practices are more likely to develop metacognitive awareness, which enables them to monitor their thinking, evaluate strategies, and adapt their approaches to problem-solving. This suggests that mathematical thinking skills not only enhance mathematical performance but also contribute to the development of broader cognitive skills.

The significance of mathematical thinking skills in STEM education and careers has also been extensively investigated. A study by National Research Council (2012) emphasized that mathematical thinking is a key competency for success in STEM fields. STEM disciplines rely on analytical reasoning, logical thinking, and the ability to apply mathematical concepts to real-world problems. Individuals with strong mathematical thinking skills are better equipped to tackle complex scientific and technological challenges and contribute to innovations in STEM fields. The study stressed the need for educational intervention as that foster mathematical thinking skills to cultivate a STEM-capable workforce.

The above explanations demonstrate the importance of having mathematical thinking skills in various domains. Mathematical thinking skills are essential for mathematical literacy, problem-solving abilities, higher-order thinking, metacognition, and success in STEM fields. Developing these skills equips individuals with the ability to make informed decisions in everyday life, approach complex problems strategically, and engage in critical and analytical thinking. Educators and policymakers should prioritize the cultivation of mathematical thinking skills to ensure individuals are prepared to meet the mathematical demands of the 21st century. The importance of having mathematical thinking skills has been widely recognized in educational research and practice. Hence, this study emphasized the need to develop mathematical thinking skills to ensure individuals are equipped to participate fully in modern society.

However, developing a deep conceptual understanding of mathematical concepts can be challenging for students. Many students rely on rote memorization without truly understanding the underlying principles, impeding their ability to apply mathematical knowledge to problem-solving situations (Khalid, Yakop & Ibrahim, 2020). Consequently, the lack of conceptual understanding hinders students' progress and may lead to misconceptions. Students often struggle to identify the appropriate problem-solving strategies and apply them correctly. Inadequate problem-solving skills can inhibit students' ability to tackle complex mathematical tasks and hinder their overall performance. Effective problem-solving skills are essential in mathematics, yet many students encounter difficulties in this area (Bobis, Anderson, Martin, & Way, 2020).

Previous researchers also found that students struggled to provide convincing explanations and justifications for their solutions, indicating a lack of proficiency in the element of convincing. Students faced challenges in identifying appropriate evidence, structuring logical arguments, and presenting their reasoning in a coherent manner. Malaysian students, on average, do not meet the international standard for mathematics performance. This could be due in part to the students' incapacity to think mathematically, resulting in difficulty in interpreting the provided situations and applying maths to generate solutions (Yunus, 2015).

The effects of the different elements of mathematical thinking on students' mathematics performance and their engagement in STEM programs have been widely studied. Each element can have unique impacts on students' learning outcomes and their interest and participation in STEM fields. Overall, challenges to mastering mathematical thinking include difficulties in assessment, prediction, teaching, and understanding the dual nature of mathematical conceptions (Sfard, 1991). Herbert (2021) argues that there is limited guidance provided by curriculum documents for assessing reasoning and difficulty in predicting critical thinking dispositions of pre-service mathematics teachers (Celik & Ozdemir, 2020). Other challenges are difficulty in teaching significant processes and reaching higher-level thinking skills (Scusa, 2008). The need for students to create and test conjectures, make sense of objects, and form arguments is also challenging (Mukuka et al., 2023).

In conclusion, the different elements of mathematical thinking can significantly impact students' mathematics performance and their engagement in STEM programs. Generalizing, conjecturing, convincing, and specializing all contribute to the development of critical thinking skills, problem-solving abilities, and a deeper understanding of mathematical concepts. Fostering these elements through targeted instruction and engaging activities can lead to improved mathematics performance and increased interest and participation in STEM discipline. Among the four elements of mathematical thinking (generalizing, conjecturing, convincing, and specializing), the most challenging element for students varies depending on individual strengths and weaknesses (Stacey & MacGregor, 2016).

Objectives of the Study

This study was conducted in which students needed to answer a mathematical thinking test to investigate their ability to understand the mathematical concept by using their mathematical thinking skills. The main objectives of the study are to examine the level of Form Four Students' Mathematical Thinking and to find out which element of mathematical thinking is the hardest to master by the students in the Malaysia context and how students differ in the ability to comprehend the mathematical thinking skill that can be explored through their answer in mathematical thinking test.

LITERATURE REVIEW

Mathematical thinking encompasses a range of cognitive processes and habits of mind that go beyond rote calculation and problem-solving. A study by Stacey and MacGregor (2016) emphasized the importance of mathematical thinking in the context of mathematical literacy. The authors argued that mathematical thinking skills are vital for individuals to navigate the complex mathematical demands of everyday life. The ability to interpret and analyze numerical information, reason quantitatively, and make informed decisions based on mathematical understanding is crucial for financial literacy, understanding statistics, and engaging with scientific and technological advancements. The concept of thinking mathematically has been explored in various studies, shedding light on different perspectives and approaches. For instance, Deliyianni et al. (2016) propose a model of thinking mathematically that encompasses four dimensions: procedural, logical, semiotic, and functional.

Understanding the concept of convincing, conjecturing, generalizing and specializing contributes to the development of instructional strategies that foster effective problem decomposition and specialized reasoning skills in students, nurture effective mathematical communication and reasoning skills in students. For instance, generalizing, as an element of mathematical thinking, involves identifying patterns, making connections, and extending mathematical concepts to broader contexts. Research has shown that students who develop strong generalizing skills demonstrate improved problem-solving abilities and a deeper understanding of mathematical concepts (Turşucu, Spandaw, & de Vries, 2020). Meanwhile, conjecturing, another element of mathematical thinking, involves making informed guesses, formulating hypotheses, and exploring mathematical relationships. Studies have shown that students who engage in conjecturing activities develop higher levels of mathematical reasoning, creativity, and critical thinking skills (Chua, 2016).

Generalizing

Generalizing in mathematical thinking refers to the process of identifying a pattern or structure within a set of mathematical objects or situations and extending it to encompass a broader range of cases. It involves looking for similarities and regularities in mathematical phenomena and formulating conjectures based on these observations. For instance, in a study by Stylianides (2019), students were presented with a series of number patterns and asked to generalize the patterns by identifying the rule that governed each sequence. The ability to generalize in this context required students to recognize the underlying relationship between the given numbers and to articulate a general formula or rule that could be applied to generate subsequent terms. Research has shown that the ability to generalize is closely linked to other aspects of mathematical thinking, such as abstraction and pattern recognition (Amit & Neria, 2008).

In a study by Amit and Neria (2008), revealed that students who demonstrated strong generalizing abilities also exhibited higher levels of abstraction, showing an understanding of the underlying structures and relationships. This highlights the interplay between generalizing and other cognitive processes involved in mathematical thinking. Generalizing is not limited to numerical patterns. It can also be applied to geometric, algebraic, and probabilistic contexts (Rivera, 2018). Thus, generalizing is a crucial element of mathematical thinking that involves recognizing patterns, making conjectures, and extending mathematical concepts to broader contexts. The ability to generalize enables individuals to identify underlying principles and apply them to solve new problems. Understanding the role of generalizing in mathematical thinking provides insights into how individuals perceive and apply mathematical concepts, promoting deeper understanding and problem-solving proficiency.

Convincing

The element of convincing in mathematical thinking involves providing evidence, justifications, and logical reasoning to support mathematical claims and arguments. This requires students to construct coherent explanations, present sound reasoning, and provide supporting evidence to establish the validity of their mathematical statements (Davies et al., 2021). Developing convincing skills requires students to think deeply about the logical coherence and validity of their mathematical arguments. Research has indicated that students who can effectively convince others of their mathematical reasoning demonstrate higher levels of conceptual understanding and improved performance on mathematical tasks (Weber, 2010).

Meanwhile, specializing, the element that involves narrowing down the focus to specific cases or instances, can also have a positive impact on students' mathematics performance and engagement in STEM. By exploring specialized instances within a broader mathematical concept or problem, students develop a deeper understanding of mathematical relationships and properties. Specializing encourages students to analyse specific cases, apply specialized techniques, and develop a more nuanced understanding of mathematical concepts (Lesh et al., 2020).

Convincing in mathematical thinking refers to the process of presenting a coherent and logical argument to validate mathematical statements, proofs, or solutions. It involves constructing clear and concise explanations, providing supporting evidence, and using deductive reasoning to establish the validity of mathematical claims (Weber, 2010). Convincing is not only limited to proof-based problems but also applicable across various mathematical domains. The ability to convince in this context required students to construct a logical argument that presented the steps and reasoning behind the theorem, demonstrating the relationship between the areas of squares on the sides of a right-angled triangle. It shows that convincing plays a central role in mathematical thinking by requiring individuals to articulate their reasoning and provide sound justifications to establish the truth of mathematical statements.

Convincing is a fundamental element of mathematical thinking that involves providing evidence, justifications, and logical reasoning to support mathematical claims and arguments. Research has shown that the ability to convince is closely related to other aspects of mathematical thinking, such as logical reasoning and critical thinking skills (Sengupta et al., 2017). Their findings revealed that students who demonstrated strong convincing abilities also exhibited higher levels of logical reasoning and critical thinking. This suggests that convincing not only promotes the ability to validate mathematical claims but also fosters deeper mathematical understanding and the development of analytical skills.

Conjecturing

Conjecturing in mathematical thinking refers to the process of formulating and testing hypotheses to explore mathematical relationships and properties. It involves recognizing patterns, making observations, and using inductive reasoning to generate conjectures that can guide further investigations (Conner et. al., 2014). The ability to conjecture in this context required students to use their understanding of numerical relationships and logical reasoning to propose potential rules or formulas that governed the sequences.

Research has shown that the ability to conjecture is closely related to other aspects of mathematical thinking, such as problem-solving skills and creativity (Karadag, 2010). Their findings revealed that students who demonstrated strong conjecturing abilities also exhibited higher levels of problem-solving skills and creative thinking. This suggests that conjecturing not only facilitates the generation of possible solutions but also promotes a deeper understanding of mathematical concepts and encourages flexible thinking.

Moreover, conjecturing is not limited to numerical or algebraic contexts but is applicable across various mathematical domains. For instance, in geometry, students can make conjectures about the properties of shapes, angles, or symmetry based on their observations and prior knowledge. This broad applicability of conjecturing highlights its importance as a universal element of mathematical thinking. The ability to conjecture enables individuals to generate initial ideas, formulate hypotheses, and guide further investigations.

Specializing

Specializing in mathematical thinking refers to the process of narrowing down the scope of a mathematical concept or problem to explore specific cases or situations. It involves examining patterns, identifying restrictions or conditions, and applying mathematical techniques to analyse and solve specialized instances (Lesh et al., 2016). Specializing is a critical element of mathematical thinking that involves narrowing down the scope of a mathematical concept or problem to focus on specific cases or instances. It is a fundamental element of mathematical thinking that involves focusing on specific cases or instances within a broader mathematical concept or problem. Over the past decade, extensive research has examined the concept of specializing as a key component of mathematical thinking (Lesh et al., 2016).

In a study by Yayuk and As'ari (2020), students engaged in tasks that required them to specialize by examining specific cases or problem variations within a broader mathematical context. The findings revealed that students who demonstrated strong specializing abilities also exhibited higher levels of problem-solving strategies and mathematical flexibility. This suggests that specializing not only helps individuals analyse specific instances but also fosters a deeper understanding of mathematical concepts and promotes adaptability in problem-solving. Moreover, specializing is not limited to particular branches of mathematics but can be applied across various domains.

RESEARCH METHOD

Research Design

A descriptive research design has been chosen as a method for this study. According to Fischer et al. (2023), quantitative research is the numerical representation and manipulation of observations for the purpose of describing and explaining the phenomena that those observations reflect. They also suggested that descriptive research is defined as a research method used to describe existing phenomena as accurately as possible, with the main objective of descriptive research is to systematically describe the existing phenomena being studied. Since this study focuses on investigating the elements of mathematical thinking skills among Form Four students, therefore, quantitative descriptive by using a mathematics test is appropriate to answer the research question.

Instrumentation

A Mathematical Thinking Test (MTT) was developed to test the respondents' mathematical thinking ability. It contains 10 questions with a total score of 45. The questions entailed problem-solving, arithmetic and basic mathematics thinking skills. Four critical elements of mathematical thinking—specialising, conjecturing, convincing, and generalising (Katagiri, 2004; Stacey, 2006)—are addressed through the 10 questions. Summarily, mathematical thinking in this study means solving a math problem by means of specialising, generalising, conjecturing and/or convincing. Following an analysis of the contents of the school's curriculum, we chose to adapt the 2012 PISA questions for the study. PISA emphasises formulating and interpreting mathematical problems to address practical difficulties, which encourages students to work on mathematically formulated problems. The components of mathematical reasoning in this study are matched with the PISA 2021 Mathematics Framework.

Besides, the components of mathematical thinking discussed by Katagiri (2004) in his paper entitled "Mathematical Thinking and How to Teach It," were also referred.

In order to ensure that the questions on the Mathematical Thinking Test (MTT) are aligned with what students in Form Four have learnt in school, mathematics curriculum for those students, which is based on the Standard Based Curriculum for Secondary Schools (KSSM) was compared. Four topics in the Form One syllabus (i.e., Measurement and Geometry, Numbers and Operations, Statistics and Probability, and Relationship and Algebra) were selected to be covered in the study's MTT test. A number of questions were gathered from previous PISA practice tests that were available on the Internet (PISA, 2012). From this pool of PISA questions, the study selected items that were judged by experts compatible with the KSSM Maths curriculum and the operationalized mathematical thinking elements. The MTT test was limited to 10 questions that covered all four elements of specialising, generalising, conjecturing and convincing. The topics and their content breakdown are listed in Table 1.

Table 1
The mapping of topics and mathematics thinking elements

Topic	Specific Content	MT Element	Question Number
Measurement and Geometry	Lines and Angles • Angles related to intersecting lines • Properties of triangles and the interior and exterior angles of triangle	Specialising	1
Numbers and Operations	Integers • Basic arithmetic operations involving integers.		4
Numbers and Operations	Integers • Basic arithmetic operations involving integers.	Generalising	2 and 5
Numbers and Operations	Integers • Basic arithmetic operations involving integers. • Positive and negative fractions.		6
Measurement and Geometry	Perimeter and Area • Perimeter		3

Statistics and Probability	Data Handling • Interpret various data representation including making inferences or prediction.	Conjecturing	9
Relationship and Algebra	Algebraic Expression • Variables and algebraic expressions. • Algebraic expressions involving basic arithmetic operations. Algebraic Formulae • Write formula based on the situation. • Change the subject formula of an algebraic equation. • Determine the value of a variable. • Solving problems involving formulae.		10
Measurement and Geometry	Perimeter and Area • Perimeter • Area • Relationship between area and perimeter	Convincing	7
Relationship and Algebra, Measurement and Geometry	Ratios, Rates and Proportions • Ratios, rates, proportions • Relationship between ratios, rates, proportions with percentages, fractions, decimals Circles • Circumference and area of a circle		8

After finalizing the questions, the instrument was distributed to the mathematics experts for content validation, especially in enhancing the instruments' level of understandability and confirming their accuracy in assessing mathematical thinking as operationalized in the study. In addition, the scoring rubric based on the answer scheme given by PISA as the final answer was also developed. The possibility of students providing their answers through the use of a diagram was also considered. The calculation below shows the example of the scoring rubric:

$$x = 180° - 51° - 41° \ \ (1\,mark)$$
$$x = 88° \ \ (1\,mark)$$

$$\measuredangle CFE = 180° - 41° - 31°$$

$$\measuredangle = 108° \ (1\,mark)$$

$$\measuredangle AFE = 180° - 108°$$

$$\measuredangle = 72° \ (1\,mark)$$

$$y = 180° - 72° - 51° \ (1\, mark)$$
$$y = 57° \ (1\, mark)$$

After constructing the scoring rubric, the math experts reviewed the work and mark allocation. They reviewed and finalised the scoring rubric based on the standard assessment procedure. Then, the answer scripts were marked using the grading rubric. Additionally, it serves as a guide for the second rater who marked 15 answer scripts as part of the inter-rater validation method. The face validity of the mathematical thinking test and inter-rater validity of the scoring rubric was done by various expertise in this field. For the mathematical thinking test, the content validation by various experts has been done multiple times to make sure the items are good enough to be used in this study. The experts' help in reviewing and validating the mathematical thinking test items, particularly in (1) refining their degree of comprehensibility and (2) determining whether they are accurate in measuring mathematical thinking as operationalized in the study. The instruments are validated by a total of five math experts and the elements in the questions have been reviewed, corrected, and commented on by three experts.

Following that, two more math specialists were chosen to recheck all of the instrument's items and prepare for actual research to test the instrument's reliability and validity. The data was analysed using Winstep software. The reliability analysis based on the Rasch Measurement Model provides valuable insights into the consistency and accuracy of the MTT Score measurements. The findings are presented in Table 2~~.~~

Table 2
Reliability Analysis Based on the Rasch Model

Statistics	**Results**	**Interpretation**
Cronbach's Alpha (MTT Score)	0.60	Good
Person Reliability	0.55	Sufficient
Item Reliability	0.97	Excellent
Person Separation	1.10	Good
Item Separation	5.96	Excellent

Cronbach's Alpha and Person Reliability indicate some room for improvement in internal consistency and precision in distinguishing between individuals, the high Item

Reliability and Item Separation values reflect the excellent performance of the items in measuring mathematical thinking and differentiating between difficulty levels. Overall, the MTT Score scale demonstrates good to excellent reliability, suggesting that it is a reliable and valid tool for assessing mathematical thinking abilities in individuals.

Sample of Study

Form Four students from a school in Segamat, Johor were selected using a purposive sampling method to achieve the study's objectives. Characteristics of the sample are; (i) the majority of the students are of average abilities, (ii) students are from a semi-urban school, and (iii) there is a mix of male and female students to achieve the objectives of the study. Students with average abilities were selected as this study aims to observe the effect of the interventions on the majority of students in the school and not only for high or low-ability students. During the study, the rubric creativity observation checklist was administered to the observers. A total of 50 Mathematical Thinking Test were distributed. And all of them were returned completely. The results were analysed quantitatively to assess the students' creativity during the interventions.

FINDINGS

To describe how students think mathematically, the researcher analysed the students' answer scripts quantitatively according to the elements of mathematical thinking. The total score of the Mathematical Thinking Test used in this study is 45 marks (100%). However, most of the respondents did not perform well with the Mean Score of the respondents was only 13.34 (SD = 5.50) and the Median was 12.50. The highest score was 25.5 marks. Detailed distributions of the respondents based on their marks and percentage of the frequency is shown in the following figure. The x-axis shows the mark from the minimum 0 to the maximum 25.5, while the y-axis shows the frequency of the scores in per cent.

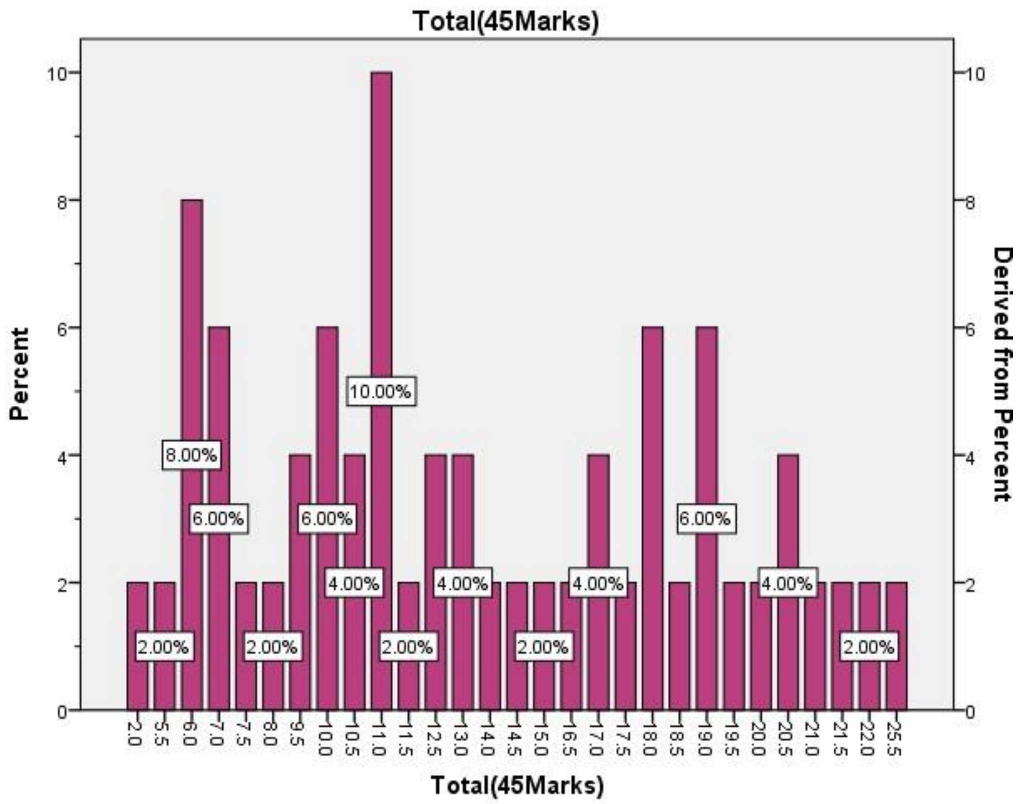

Figure 1: The Distribution of the Marks and the Percentage of the Frequency

Referring to Figure 1, 2% of the respondents scored the lowest two marks and 2% of the respondents scored the highest 25.5 marks. The students were then categorized into 3 categories of low, average, and high, based on the mean and standard deviations (SD = 5.50). Students with scores below 1 standard deviation away from the mean will be categorized into the low group. Those between ± 1 standard deviation from the mean will be categorized as average and those who scored higher than 1 standard deviation away will be categorized as high scorers.

Table 3

Students' Performance by Levels

Levels	Explanation	Score Range
Low	Those who scored below (13.34 - 5.50) = 7.84	0 to 8
Average	Those who scored between 5.50 and (13.34 + 5.50) = 18.84	9 to 19
High	Those who scored more than (13.34 + 5.50) = 18.84	19.5 to 25.5

Later, based on the range of categories, the respondents were grouped into three levels namely, 1 = Low (0 to 8), 2 = Average (9 to 19) and 3 = High (20 to 25.5). Table 4 shows the respondents' distribution according to the score levels.

Table 4
Percentage of Students' Performance by Levels

Levels	Percent	Cumulative Percent
Low	22	22
Average	62	84
High	16	100

Table 4 shows the percentage of the respondents according to the levels. 22% of the respondents are at the Low level, 62% at the Average level and only 16% of them are at the High level.

Table 5
Summary of the MTT Result

MT Element	Question	Frequency			Percentage %		
		Right	Wrong	Incomplete	Right	Wrong	Incomplete
Specialising	1a	27	23	0	54	46	0
	1b	15	32	2	30	64	6
	4	9	14	27	18	28	54
Generalising	2	32	1	17	64	2	34
	5	24	24	2	48	48	4
	6	0	19	31	0	38	62
Conjecturing	3	1	26	23	2	52	46
	9a	9	41	0	18	82	0
	9b	1	49	0	2	98	0
	10	7	41	2	14	82	4
Convincing	7a	8	9	33	16	18	66
	7b	0	49	1	0	98	2
	7c	8	35	7	16	70	14
	8	0	37	13	0	74	26

Students' answers to the items of the test are divided into three parts, including Right Answer, Incomplete Answer and Wrong Answer. Descriptive statistics was used to analyse the data, such as frequency and percentage. Table 5 presents the frequencies and percentages of two types of answers given by the participants in this study.

Analysis of Specialising Element

There are two questions for the Specialising element in the test. Question 1 is divided into two parts (part a and part b), which carry 5 marks in total (Question 1a carries 2 marks and Question 1b carries 3 marks). While Question 4 carries 3 marks in total.

Question 1 is about Lines and Angles. The students need to know the angle related to intersecting lines, the properties of triangles and the interior and exterior of triangles. While Question 4 is about the basic arithmetic operations involving integers by using a simple problem-solving situation. It carries 3 marks in total. The marks indicated the students' mathematical thinking skills in specialising elements. Specializing in this question takes place when students are involved the process of narrowing down the scope of a mathematical concept or problem to explore specific cases or situations. It involves examining patterns, identifying restrictions or conditions, and applying mathematical techniques to analyse and solve specialized examples. The details of the question can be referred in the Appendix section.

From Table 5, 54% of the students managed to answer the Question 1a. Unfortunately, only 30% of the total number of students had the ability to answer Question 1b correctly. The result shown depicts that the students have low specialising skill in answering mathematical thinking test. We can conclude that 58% of the total number of students have low specialising skills. Only 9 out of 50 students have this element of mathematical thinking skill. The ability to specialize is important for the students, which will enable them to break down complex problems, apply specialized techniques, and analyse specific situations within a broader mathematical context.

Analysis of Generalising Element

There are three questions which assessed the generalising element in the test which carry 14 marks in total (Question 2 carries 4 marks, Question 5 carries 4 marks, and Question 6 carries 6 marks). All questions covered Numbers and Operations topic, which involved basic arithmetic operations involving integers and fractions. Generalizing in mathematical thinking refers to the process of identifying a pattern or structure within a set of mathematical objects or situations and extending it to encompass a broader range of cases. All questions can be referred in the Appendix section.

From Table 5, 64% of the total number of students could answer the question which means that more than half of them have the ability to generalise in that particular question. Only one student could not figure the question out. Meanwhile, for Question 5, the lowest and highest number of students who have generalising element in their mathematical thinking skill shared the same percentage which is 48%. On the other hand, we can see that 52% of the students had low generalising ability in this question. However, 24 students managed to answer the question perfectly.

Surprisingly, in Question 6, no students managed to answer the question completely. The highest mark for this question is only 3 and only one student got the answer correct. This depicts the students could not generalise the question given. Half of them got below than 1 mark out of 6 marks. The results shown 98% of the students got 2 marks and below.

Analysis on Conjecturing Element

There are three questions that assessed students' conjecturing element of mathematical thinking skill. Question 3 consists of 4 marks, Question 2 consists of 2 marks (Question 1a carries 1 mark, and Question 1b carries 1 mark), and Question 10 carries 4 marks which in total of 10 marks for all those three questions. Question 3 needed the students to find the perimeter, while Question 9 assessed the students' interpretation of the data representation, and lastly for Question 10, the students needed to use their skill in relating the algebraic expression and algebraic formulae. The example of the questions can be referred in the Table 1. Conjecturing, another element of mathematical thinking, involves making informed guesses, formulating hypotheses, and exploring mathematical relationships.

From the result shown, 84% of students did not manage to get the average mark, which inferred that they have low conjecturing skills. Only 2% of the students have the conjecturing skill in Question 3, while 12% have the average conjecturing element of mathematical thinking skill. For Question 9, the percentage of the students who did not get any marks is above 80%. It can be seen clearly that students did not grasp this element of mathematical thinking skill.

Lastly, the highest mark for Question 10 is 4 out of 6 in total which only be achieved by 7 students out of 50. Most of the students got 0 marks which resulted by 82% of them have low ability in this conjecturing element.

Analysis on Convincing Element

This element has been assessed by using only two questions, which are Question 7 and Question 8. For Question 7, it comprised of 6 marks, but it was divided by three sub-questions and they carried 2 marks for each sub-question. It determined the students' convincing skill in calculating the perimeter and area. While Question 8 consisted of 4 marks in total. This question asked the students to relate all concepts in Relationship and Algebra, Measurement and Geometry. This question assessed their concept of Ratios, Rates and Proportions as well as Circles. Convincing in mathematical thinking refers to the process of presenting a coherent and logical argument to validate mathematical statements, proofs, or solutions. It involves constructing clear and concise explanations, providing supporting evidence, and using deductive reasoning to establish the validity of mathematical claims.

From Table 5, we can see clearly that most of the students have below than the average score of Question 7. More than 50% of the students did not have the convincing element in mathematical thinking skill. As we can see from the table, 84% of the students could not answer Question 7a, 100% of the students got 1 mark and below, while 84% of the students answered wrongly in Question 7c. For question 8, none of them answered it correctly. The highest mark was only 1.5 out of 4 marks and only 26% of the total number of students who got the idea to solve the question.

DISCUSSION

Data for this research consist of students' mathematical thinking test score. The students' mathematical thinking test scores are classified into three categories which are low, average, and high. By using the descriptive analysis, the respondents' distribution of the marks was explored. The highest score is 25.5 marks out of 43 marks. The Mean Score of the respondents was 13.34 (SD = 5.50) and the Median is 12.50. The finding shows that most of the respondents did not perform well. There was 62% of the respondents at Average Level, 22% at the Low Level and another 16% was at High Level. This means that majority of the respondents (84%) was at the Moderate and Low Levels. It demonstrates that more than half students have relatively low mathematical thinking skill.

Based on the findings, students are more capable when solving generalising test items. These items request students to look for patterns and relationships from the questions given. The generalising element is an aspect of mathematical thinking skill that is naturally owned by every person. It is parallel with Gestalt's cognitive psychology theory who mentions that every person has a tendency to complete and to fill incomplete experiences to make them meaningful (Guberman, 2015). The Form Four students at the secondary school are those who have already been familiar with the material of basic arithmetic operations involving integers since they were in elementary school, and the more complicated version of this material is relearned in high school. By doing so, when there is a recalling act of that material, the level of difficulty faced by the students will be lower compared with other types of test items.

On the other hand, the most challenging element in mathematical thinking skills in this study is conjecturing. Despite that, conjecturing also has lower marks among those four elements of mathematical thinking in this study, contradicted by the previous study, which reported that students often face difficulties in constructing convincing elements (Sengupta et al., 2017).

Another line of research has explored the link between mathematical thinking skills and problem-solving abilities. Sengupta et al. (2017) conducted a meta-analysis of studies investigating the impact of mathematical thinking instruction on problem-solving performance. The findings revealed a positive relationship between mathematical thinking skills and problem-solving outcomes across various educational contexts and age groups. The authors concluded that developing mathematical thinking skills enhances individuals' capacity to approach complex problems, identify patterns and relationships, and devise effective strategies for problem-solving. Therefore, the respondents were not able to perform well in this test due to the questions in this test are mostly problem-based questions.

CONCLUSION

In conclusion, it can be approved that the majority of the respondents or the participants have low ability and performance in the Mathematical Thinking Test. This is consistent with other findings in previous research indicating that students' mathematical thinking skills are quite inadequate. Many participants were unable to answer even the most basic questions on the test, therefore, teachers and other relevant stakeholders should pay closer attention to this issue. This is consistent with numerous findings in previous studies that stated mathematics is very challenging for many students. The findings provide a basis for other researchers to use the MTT instrument in assessing mathematical thinking abilities among Form Four or tenth-grade students in other backgrounds.

The MTT instrument was developed and tested using the Rasch Measurement Model, which is a newly developed tool or instrument that was created using mathematical thinking elements and appropriate procedures. It can be modified to fit the needs of any country's mathematics education system. The analysis of the Rasch Measurement Model, which was used to evaluate the validity and reliability of the MTT instrument, shows that this instrument is highly valid and reliable for measuring mathematical thinking abilities. This study established the instrument's validity and reliability in assessing mathematical thinking. Hence, it can be replicable.

This study suggests that it is necessary to find solutions to overcome student difficulties in learning mathematics, for instance, by doing fun activities in the classroom that can foster their mathematical thinking skills or a constructivist learning environment. For further research, it is recommended to add a wider range of students' abilities in future. It is recommended to take samples across Malaysia, including Sabah and Sarawak to represent the whole 10th grade Malaysian population and to get a better representation of the students' performance in mathematical thinking. It is also recommended that the Malaysian Ministry of Education give more training to mathematics teachers to implement the specific approaches in the classroom effectively. Thus, it is easier for the school management or respective authorities to monitor the teaching progress and to find solutions to any problem that might occur in future.

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REFERENCES

Alcock, L., & Weber, K. (2010). Referential and syntactic approaches to proving: Case studies from a transition-to-proof course. *Research in collegiate mathematics education VII*, 7, 93.

Amit, M., & Neria, D. (2008). "Rising to the challenge": Using generalization in pattern problems to unearth the algebraic skills of talented pre-algebra students. *ZDM*, 40, 111129.

Atmowardoyo, H. (2018). *Research methods in TEFL studies: Descriptive research, case study, error analysis, and R & D. Journal of Language Teaching and Research, 9(1)*, 197-204.

Bakker, A., Cai, J., & Zenger, L. (2021). Future themes of mathematics education research: An international survey before and during the pandemic. Educational Studies in Mathematics, 107(1), 1-24.

Bobis, J., Khosronejad, M., Way, J., & Anderson, J. (2020). "Sage on the stage" or "meddler in the middle": shifting mathematics teachers' identities to support student engagement. Journal of Mathematics Teacher Education, 23(6), 615-632.

Calder, N., Jafri, M., & Guo, L. (2021). Mathematics education students' experiences during lockdown: Managing collaboration in eLearning. *Education Sciences*, *11(4)*, 191.

Celik, H. C., & Ozdemir, F. (2020). Mathematical Thinking as a Predictor of Critical Thinking Dispositions of Pre-Service Mathematics Teachers. International Journal of Progressive Education, 16(4), 81-98.

Chua, B. L. (2016). Justification in Singapore secondary mathematics. In *Developing 21st Century Competencies In The Mathematics Classroom: Yearbook 2016, Association of Mathematics Educators* (pp. 165-187).

Conner, A., Singletary, L. M., Smith, R. C., Wagner, P. A., & Francisco, R. T. (2014). Identifying kinds of reasoning in collective argumentation. *Mathematical Thinking and Learning*, 16(3), 181-200.

Creswell, J. W. (2012). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research*. Pearson Education, Inc.

Davies, B., Alcock, L., & Jones, I. (2021). What do mathematicians mean by proof? A comparative-judgement study of students' and mathematicians' views. *The Journal of Mathematical Behavior*, *61*, 100824.

Deliyianni, E., Gagatsis, A., Elia, I., & Panaoura, A. (2016). Representational flexibility and problem-solving ability in fraction and decimal number addition: A structural model. *International Journal of Science and Mathematics Education*, 14, 397-417.

Dubinsky, E., Mcdonald, M.A. (2001). APOS: A Constructivist Theory of Learning in Undergraduate Mathematics Education Research. In: Holton, D., Artigue, M., Kirchgräber, U., Hillel, J., Niss, M., Schoenfeld, A. (eds) The Teaching and Learning of Mathematics at University Level. *New ICMI Study Series*, vol 7. Springer, Dordrecht. https://doi.org/10.1007/0-306-47231-7_25

Fischer, H. E., Boone, W. J., & Neumann, K. (2023). Quantitative research designs and approaches. In *Handbook of research on science education* (pp. 28-59). Routledge.

Herbert, S. (2021). Overcoming challenges in assessing mathematical reasoning. Australian Journal of Teacher Education (Online), 46(8), 17-30.

Ilyas, M., Meiyani, E., Ma'rufi, M. R., & Kaewhanam, P. (2022, October). Improving students' ability in learning mathematics by using the science, technology, engineering, and mathematics (STEM) approach. *In Frontiers in Education* (*Vol. 7,* p. 966687). Frontiers Media SA.

Karadag, Z. (2010). *Analyzing students' mathematical thinking in technology-supported environments* (Doctoral dissertation, University of Toronto).

Katagiri, S. (2004). *Mathematical thinking and how to teach it*. CRICED, University of Tsukuba.

Khalid, M., Yakop, F. H., & Ibrahim, H. (2020). Year 7 students' interpretation of letters and symbols in solving routine algebraic problems. The Qualitative Report, 25(11), 41674181.

Khalil, M. (2019). Concept process with mathematical thinking tools under the domain of Piaget's theory of cognitive development. JCTE, 3.

Lesh, R., Yoon, C., & Zawojewski, J. (2020). John Dewey revisited—making mathematics practical versus making practice mathematical. *In Foundations for the future in mathematics education* (pp. 315-348). Routledge.

Mukuka, A., Balimuttajjo, S., & Mutarutinya, V. (2023). Teacher efforts towards the development of students' mathematical reasoning skills. Heliyon, 9(4).

National Research Council. (2012). *Education for STEM literacy: Thinking critically in a digital age*. National Academies Press.

PISA, O. (2012). Results in focus 2014-02-17. http://www,oecd. org/pisa/keyfindings/ pisa2012-results-o-verview,pdf.

Rivera, F. D. (2018). Pattern generalization processing of elementary students: Cognitive factors affecting the development of exact mathematical structures. *Eurasia Journal of Mathematics, Science and Technology Education*, 14(9), em1586.

Schoenfeld, A. H. (2016). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics (Reprint). Journal of education, 196(2), 1-38.

Scusa, T. (2008). *Five processes of mathematical thinking*.

Sengupta, P., Jonassen, D. H., & Lee, C. B. (2017). Effects of teaching for mathematical thinking on students' problem-solving skills: A meta-analysis. *Journal for Research in Mathematics Education, 48(2)*, 165-198.

Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. Educational studies in mathematics, 22(1), 1-36.

Stacey, K. (2006). What is mathematical thinking and why is it important. *Progress report of the APEC project: collaborative studies on innovations for teaching and learning mathematics in different cultures (II)—Lesson study focusing on mathematical thinking*.

Stacey, K., & MacGregor, M. (2016). Mathematical thinking: Why, what, and how. *ZDM Mathematics Education, 48(3)*, 327-334.

Stylianides, A. J. (2019). Understanding and characterizing student reasoning in mathematics classrooms. *ZDM Mathematics Education, 51(1),* 1-16.

Sukamolson, S. (2007). *Fundamentals of quantitative research. Language Institute Chulalongkorn University, 1(3)*, 1-20.

Swidan, O., & Tall, D. (2019). Metacognition and mathematical thinking. *ZDM Mathematics Education, 51(3)*, 429-441.

Turşucu, S., Spandaw, J., & de Vries, M. J. (2020). *The effectiveness of activation of prior mathematical knowledge during problem-solving in physics*.

Weber, K. (2010). Mathematics majors' perceptions of conviction, validity, and proof. *Mathematical thinking and learning*, 12(4), 306-336.

Yayuk, E., & As' ari, A. R. (2020). Primary School Students' Creative Thinking Skills in Mathematics Problem Solving. *European Journal of Educational Research, 9(3)*, 12811295.

Yunus, A. S. M. (2015). Developing Students' Mathematical Thinking: How Far Have We Come? Penerbit Universiti Putra Malaysia.