Developing a Hypothetical Learning Trajectory for Translation in Geometry Using the Chess Game Context

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Abstract

Students often find it difficult to learn the concept of translation because it is abstract and too procedural. This study aims to develop a learning trajectory (HLT) for geometry translation material with a chess game context. This study uses a design research method consisting of three stages, namely preliminary design, design experiment, and retrospective analysis. The learning path developed consists of four main activities: observing chess game videos and pawn movements as an introduction to the context, representing pawn movements on a Cartesian coordinate plane, identifying the characteristics of translation transformations, and discovering the formulas and concepts of translation. Based on the implementation results, this HLT can facilitate the development of students' understanding from symbolic to formal forms. Students can also convert pawn movements into a coordinate system and discover patterns of relationships that lead to the formulation of translation formulas. Additionally, the use of the chess game context has proven to enhance students' interest and engagement in learning. These findings indicate the potential of context-based HLT to function as a situational learning model that can apply geometric concepts.

Keywords: *Chess; Hypothetical Learning Trajectory; Realistic Mathematics Education; Translation.*

Abstrak

Siswa sering merasa kesulitan dalam mempelajari konsep translasi karena bersifat abstrak dan terlalu prosedural. Penelitian ini bertujuan untuk mengembangkan lintasan pembelajaran (HLT) untuk materi translasi geometri dengan konteks permainan catur. Penelitian ini menggunakan metode penelitian desain yang terdiri dari tiga tahap, yaitu preliminary design, design experiment, dan retrospective analysis. Lintasan belajar yang dikembangkan tersusun dari empat aktivitas utama yaitu mengamati video permainan catur dan pergerakan pion sebagai pengenalan konteks, merepresentasikan pergerakan pion pada bidang koordinat Kartesius, mengidentifikasi karakteristik transformasi translasi, dan menemukan rumus serta konsep dari translasi. Berdasarkan hasil implementasi, HLT ini mampu memfasilitasi perkembangan pemahaman siswa dari bentuk simbolik menuju bentuk formal. Siswa juga dapat mengubah pergerakan pion ke sistem koordinat dan menemukan pola hubungan yang mengarah pada penyusunan rumus translasi. Selain itu, penggunaan konteks permainan catur terbukti meningkatkan minat dan keterlibatan siswa dalam pembelajaran. Temuan ini menunjukkan potensi HLT berbasis konteks dapat berfungsi sebagai model pembelajaran situasional yang dapat diterapkan konsep geometri.

Kata Kunci: Catur; Lintasan Belajar Hipotetik; Pendidikan Matematika Realistik; Translasi.

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INTRODUCTION

At school, mathematics is considered one of the most important subjects. However, many students still find mathematics difficult, especially when the material being studied is abstract, such as geometric transformations. This is not only because the material is complex. The way teachers teach, which involves directly presenting the concepts and providing exercises to practice calculation skills and the use of formulas on previously presented concepts (Rangkuti & Siregar, 2020), means that students' understanding of geometric transformations only involves the physical movement of geometric shapes rather than the mapping of points on a flat plane (Uygun, 2020). This results in students expending too many cognitive resources on information that is irrelevant to what they are learning, leading them to experience cognitive overload, which reduces the effectiveness of their learning (Liu et al., 2023). In addition, students also have difficulty developing the meaning of translation due to a lack of geometric understanding (Sahara et al., 2023).

This situation is exacerbated by the lack of visual aids in the learning process. Material is still delivered verbally without involving media and activities that support visualization. This makes it difficult for students to imagine the objects they want and rely solely on instructions from the teacher (Yunianto et al., 2024). Meanwhile, the use of visual aids enables students to visualize situations and reinterpret geometric transformation concepts (Sahara et al., 2023). This is supported by research showing that the use of visual situations can help students significantly improve their visual memory thanks to the dynamic nature of such tools (Sosa & Aguilar, 2021).

The impact of this meaningless learning not only hinders conceptual understanding but also affects students' attitudes toward mathematics. When students do not understand the material, they lose confidence and interest in learning. Over time, this creates a negative perception that mathematics is a difficult subject because it is often associated with numbers, formulas, and theorems, making it seem less accessible and less meaningful (Rangkuti & Siregar, 2020). Studies show that by using games, students can build an understanding of new concepts because they can connect their existing knowledge and experiences with new knowledge (Wijaya & Doorman, 2021). The appropriate use of gamification can create a rich and satisfying environment where students develop a love for learning in subjects such as mathematics and increase their motivation to learn in that subject (Hammadi et al., 2024).

The Realistic Mathematics Education (RME) approach can be one way to address current issues. This approach can bridge the gap between abstract mathematical concepts and students' everyday experiences (Adha et al., 2024). Additionally, RME can be used as a framework to develop students' mental understanding from concrete to more abstract levels according to their level of comprehension (Rawani et al., 2023). By using real-life examples, this approach can help students actively build their own understanding. In line with this, within the RME context, students are encouraged to explore and understand mathematical concepts by observing how these concepts can be applied in everyday situations, enabling them to recognize the relevance and utility of mathematics in their lives (Andzin et al., 2024).

Several studies have shown that visual and local cultural contexts have been used effectively to introduce mathematical concepts, one of which is geometric transformation. Some studies have found that the use of South Sumatran dance contexts and Cartesian coordinate models of floor tiles can help students understand the concept of geometric transformation (Rawani et al., 2023). The use of batik motifs also helps students develop ideas and strategies for identifying main patterns, but also allows them to understand visual geometric object transformations (Sahara et al., 2023). This demonstrates that geometric concepts are connected to students' real-life experiences. This aligns with the RME principle that mathematics is a human activity.

However, no research has systematically developed a learning trajectory (HLT) for geometric translation within the context of strategic games like chess. Previous studies have primarily focused on cultural contexts but have not optimized the potential of pattern-based movement games like chess, which have direct relevance to geometric transformations (Giouvantsioudis, 2024; Sosa &

Aguilar, 2021). In this context, chess becomes a powerful tool for use as a learning context. The movement patterns of each piece in chess follow fixed rules and are easily modeled in a coordinate system, which naturally relates to geometric transformations (Nadarajan et al., 2023; Wijaya & Doorman, 2021).

In addition to supporting cognitive development, the game-based approach also has a positive impact on students' affective and motivational aspects in learning mathematics (Hammadi et al., 2024; Liu et al., 2023). When students feel happy and interested, they tend to be more active in exploring and understanding the concepts being taught. This meaningful and enjoyable learning experience is the fundamental principle of the RME approach (Sahara et al., 2023).

However, little is known about how strategic games like chess can serve as a foundation for developing a structured learning pathway for geometric translation. Such game-based approaches open new possibilities in designing learning trajectories that allow students to build concepts gradually and contextually. Therefore, this study aims to develop a Hypothetical Learning Trajectory (HLT) for geometry translation learning using the context of chess games.

RESEARCH METHODS

This study used the design research method. This research consists of three different stages, namely (1) preliminary design, (2) design experiment, and (3) retrospective analysis.

Preliminary Design

At this stage, the research activities focused on developing a Hypothetical Learning Trajectory (HLT) for translational geometry learning. The HLT designed aims to map the learning flow while predicting the development of students' understanding in mastering the concept of translation. The researcher developed a learning design that integrates the context of the chess game as a concrete medium for understanding geometry transformation. The learning trajectory design is outlined in the form of Hypothetical Learning Trajectory (HLT) as shown in Figure 1.

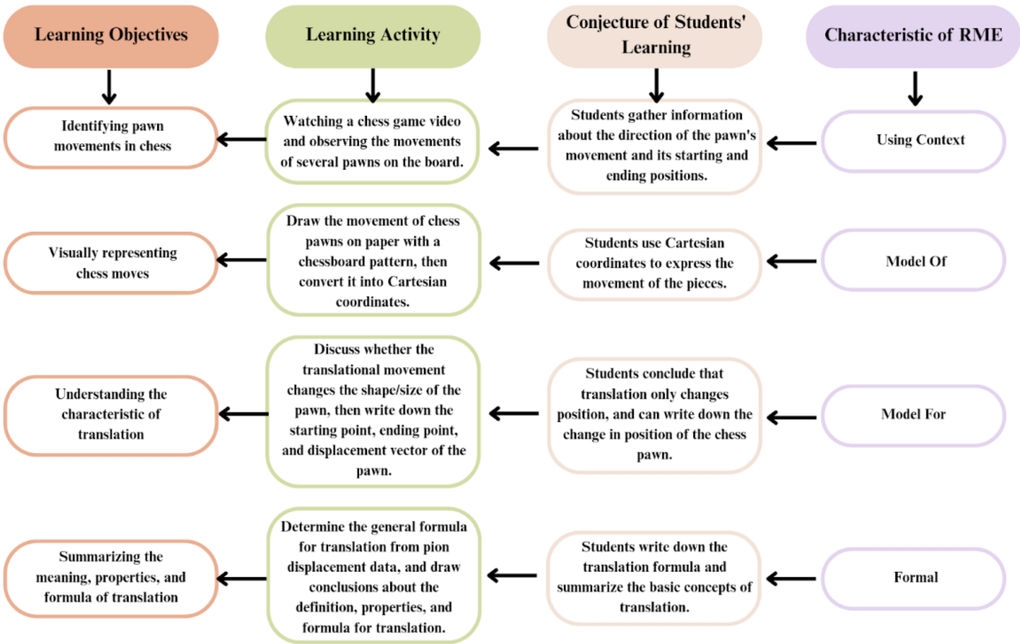


Figure 1. Design of the Hypothetical Learning Trajectory (HLT) for translation material

Design Experiment

In the teaching experiment stage, the chess-based learning design that had been developed was implemented in a real classroom setting. This experimental stage was crucial for testing the validity of the HLT that had been designed previously, as well as evaluating the suitability between the predicted development of student understanding and the actual learning outcomes.

Retrospective Analysis

At this stage, data from the implementation results are further examined to determine how effective the designed learning design is. This analysis focuses on three elements, namely learning objectives, improvement in student understanding, and the validity of the HLT design that has been developed previously. Here, the focus is on presenting the results of the experiment by providing a detailed description of the HLT testing in small groups.

The subjects of this study consisted of six eighth-grade students from a junior high school in Surabaya. These six students were categorized into three ability levels, namely high, medium, and low, with each level represented by two students. The determination of these ability levels was based on teacher recommendations and their learning outcomes in previous learning materials. By selecting three different ability levels, this study was able to represent the entire range of abilities in the class. This shows how this learning approach can affect students with varying ability levels. In addition, the data collection techniques used included interviews to explore students' conceptual understanding of the concept of translation, as well as analysis of students' worksheets as physical evidence of their learning progress.

RESULTS AND DISCUSSION

This study produced a learning path for the concept of translation designed to help students build understanding gradually through the context of chess. The path consists of four main activities, including: (1) observing chess game videos and pawn movements as an introduction to the context; (2) representing pawn movements on a Cartesian coordinate plane; (3) identifying the characteristics of translation transformations; and (4) discovering the formulas and concepts of translation. The implementation of each activity designed in the resulting learning path will be further elaborated below.

Activity 1: Students watch a video of a chess game and observe the movements of the chess pawns

To establish a concrete context for translation learning, students are introduced to chess through video media. The following image shows the video display that students observe as a starting point for learning.

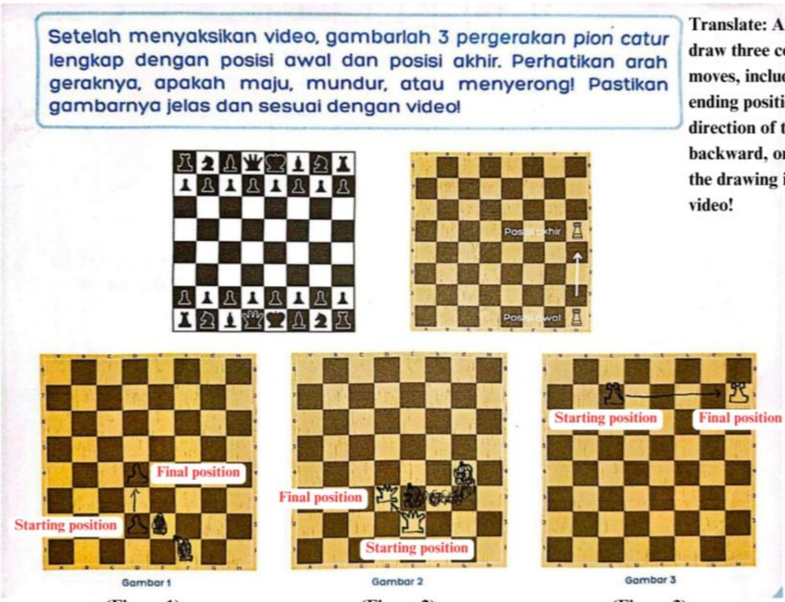
Translate: Do you know about the game of chess? Have you ever played it? Let's watch the following chess game video!

Figure 2. Chess Game Video

In activity 1, students will be introduced to one of the real-life contexts that will be discussed, namely chess. First, students are asked to scan a barcode that will direct them to a YouTube video about chess. Students must then observe each move of the chess pieces shown in the video. Students will gather information about chess and how the pieces move in the game. The use of YouTube videos is intended to make students interested in the learning activities that will be carried out.

Activity 2: Students describe the movement of chess pawns on a Cartesian coordinate system

After watching the video, students were asked to identify three chess pawn moves that they saw and draw them on the worksheet provided. This activity helped students translate their visual understanding into a more systematic form of representation. The following images are the students' visualizations of the pawn moves they observed.



Translate: After watching the video, draw three complete chess pawn moves, including the starting and ending positions. Pay attention to the direction of the move is it forward, backward, or sideways? Make sure the drawing is clear and matches the video!

(Figure 1) (Figure 2) (Figure 3)

Figure 3. Students' Answers Related to the Chess Movement Images in the Video

Figure 3 shows that students were able to understand and identify the rules of chess pawn movement as context. Students could determine the initial and final positions of the pawn after it had been moved. Students can also describe the direction of the chess pawn's movement. This serves as the initial foundation for students to describe the pawn's position on the Cartesian coordinate system. This is because the chessboard can serve as a tool to understand the coordinate system. The grid pattern of columns and rows on the chessboard visually and tangibly represents the X-axis and Y-axis.

Furthermore, students are asked to convert the drawings that have been made into cartesian coordinates. The following figure illustrates the students' representation of the pawn position in the form of coordinate pairs.

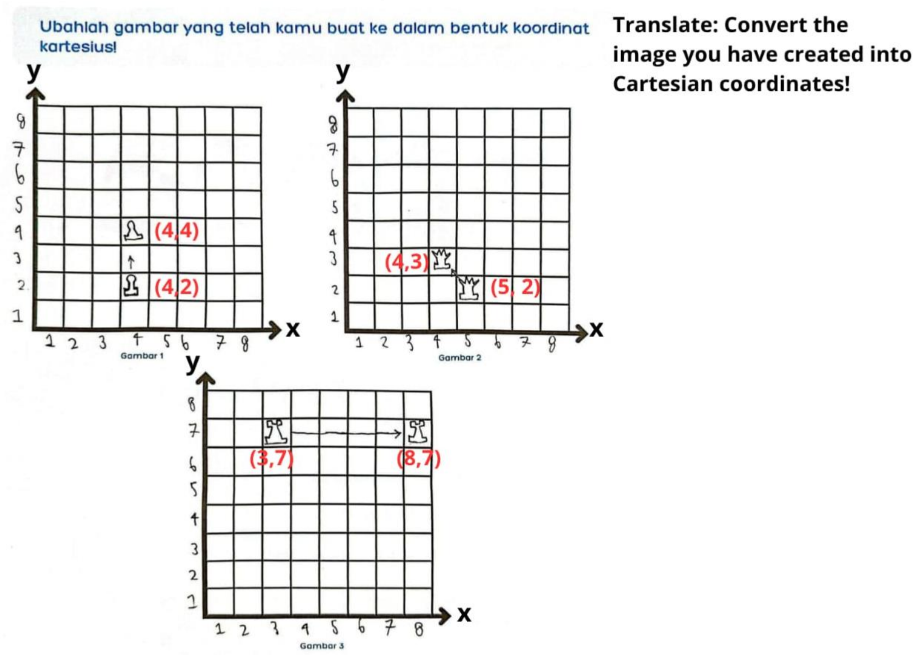
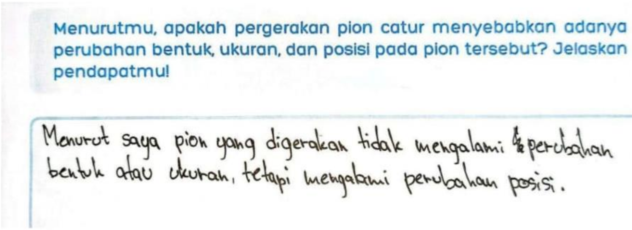


Figure 4. Students' Answers Regarding the Movement of Pawns on the Cartesian Coordinate System

Based on Figure 4, it can be seen that students can accurately determine and write down the starting point and end point of the pawn's movement. This shows the students' understanding of describing the position of a chess pawn in Cartesian coordinates. As in the coordinate system in picture 1, students place the starting point of the pawn at (4,2) and end at point (4,4), which shows a vertical shift. In the coordinate system in picture 2, the movement is written as (5,2) to (4,3), indicating a diagonal shift. And in the coordinate system in picture 3, it depicts a horizontal shift from the coordinate point (3,7) to (8,7). This indicates that students are not only able to observe changes in position but also successfully convert the pawn's position from the chessboard image to the Cartesian coordinate system accurately and precisely.

Activity 3: Students identify the characteristics of translation

Activity 3 focuses on students' conceptual understanding of translational characteristics. Here, students identify the nature of translation and the shifting pattern of the chess pawns. The following figure shows the results of students' thinking in concluding the characteristics of translation.

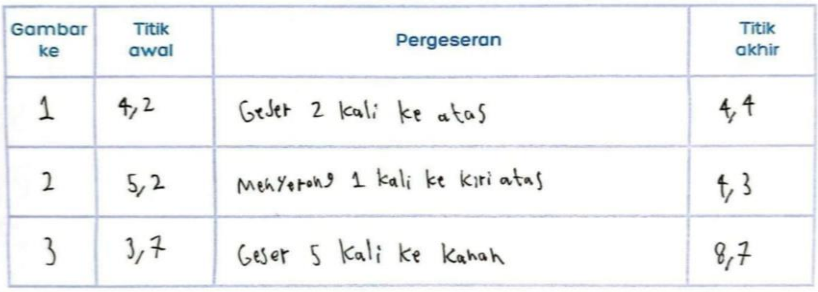

Menurutmu, apakah pergerakan pion catur menyebabkan adanya perubahan bentuk, ukuran, dan posisi pada pion tersebut? Jelaskan pendapatmu!

Menurut saya pion yang digerakan tidak mengalami perubahan bentuk atau ukuran, tetapi mengalami perubahan posisi.

Translate:
In your opinion, does the movement of a chess pawn cause changes in its shape, size, and position? Explain your opinion!

Answer: In my opinion, the pawn that is moved does not undergo any change in shape or size, but only undergoes a change in position.

Figure 5. Students' Answers Regarding the Properties of Translation



Gambar ke	Titik awal	Pergeseran	Titik akhir
1	4,2	Geser 2 kali ke atas	4,4
2	5,2	Menyerong 1 kali ke kiri atas	4,3
3	3,7	Geser 5 kali ke kanan	8,7

Picture No.	Starting poin	Translation	Ending poin
1.	(4,2)	Shift 2 units up	(4,4)
2.	(5,2)	Diagonally shift 1 unit to upper left	(4,3)
3.	(3,7)	Shift 5 units to the right	(8,7)

Figure 6. Students' Answers Regarding Pawn Movement

Based on Figure 5, students can conclude that translation only causes a change in the position of an object without changing its shape or size. This is supported by the following interview results.

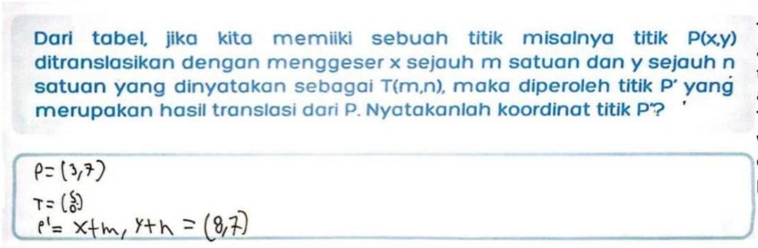
Researcher : "Why did you answer that the chess pawn only undergoes a change in position?"

Student : "I think that if I move the pawn forward, it will only change its position forward and its size and shape will not change."

Additionally, students were able to explain the movement of the pawn using a pair of coordinates that describe the initial and final positions, while also stating the direction of the shift. Based on Figure 6, it can be seen that in picture 1, the initial position of the pawn is (4,2), which is then shifted upward by 2 units to (4,4). In picture 2, the movement from the initial position (5,2) is then shifted one unit to the left and one unit upward to (4,3). And in picture 3, the change is from (3,7) then shifted five units to the right to (8,7). The process of identifying the initial and final point pairs, as well as the ability to express the shift in coordinate form, reflects a shift in students' thinking from merely observing motion (motion view) to understanding translation as a systematic mapping in the coordinate plane (mapping view) (Akarsu, 2022).

Activity 4: Students discover the formula and basic concepts of translation

After going through several activities, starting from observing the movement of pawns, representing them in coordinates, and recognizing the properties of translation, students are then directed to make generalizations based on the patterns they observe. From activity 3, students obtain information about the relationship between the initial position, displacement, and final position of the pawn, which can help students formulate a general translation formula. The following image shows the results of students in formulating the general form of translation.



Translate: From the table, if we have a point, for example point P(x,y), translated by shifting x by m units and y by n units, expressed as T(m,n), then we obtain point P', which is the result of the translation of P. Express the coordinates of point P'!

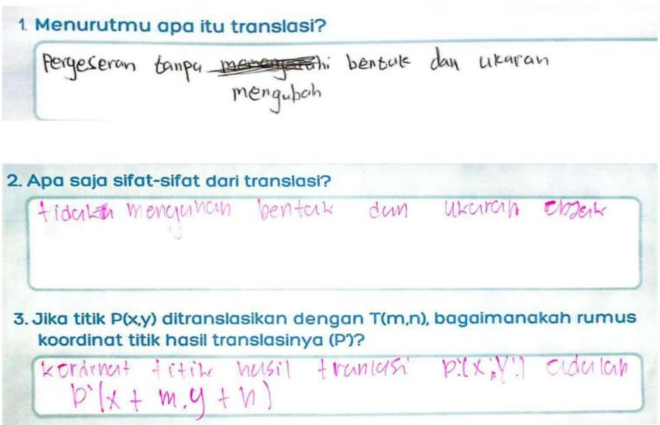
Figure 7. Students' Answers in Finding the Translation Formula

From Figure 7, it can be seen that students can determine the general formula for translation. In determining the general formula, students take one point from the movement of the chess pawn and generalize it into a formal formula. This is confirmed by the following interview results.

Researcher : "How did you find the formula (x + m, y + n)?"

Student : "I first wrote down the starting point. Then I looked at the third shift table, where the shift is moving 5 units to the right, which means changing the position on the x-axis. So, from point (3,7) to (8,7), we get (3 + 5, 7 + 0). Since the starting point is (x,y), the final point is (x + m, y + n)."

In addition to finding the general formula for translation, students also wrote conclusions related to the basic concepts of translation. This shows students' understanding of the concepts they have learned. The following image shows students' statements regarding the basic concepts of translation.



Translate:
1. What do you think translation is?

Answer: Displacement without changing shape and size.

Translate:
2. What are the characteristics of translation?

Answer: Does not change the shape and size of objects.

Translate:
3. If point P(x,y) is translated by T(m,n), what is the formula for the coordinates of the translated point (P')?

Answer: The coordinates of the translated point P (x,y) are P' (x+m, y + n)

Figure 8. Students' Answers Related to the Basic Concept of Translation

Based on Figure 8, students conclude that translation is the shifting of an object without changing its shape and size. The nature of translation is that it does not change the shape and size of an object. And if there is a point (x, y) that is then translated to (m, n), the resulting coordinates of the translation are (x + m, y + n).

The results of the learning trajectory implementation show that the chess-based approach is effective in helping students gradually build an understanding of the concept of translation. Activities in HLT encourage visual and symbolic exploration, so that students understand the meaning of object shifts in a coordinate system, rather than simply memorizing (Fauzan, 2024). The process of generalizing the translation formula reflects a shift in thinking from observation to formal form, marking the success of the HLT design (Avcu & Çetinkaya, 2021). This aligns with the principle that HLT shapes students' thinking structures through systematic and contextual activities (Rangkuti & Siregar, 2020), and is reinforced by findings that HLT designed according to students' cognitive development is more effective than direct approaches (Baroody et al., 2022).

Although most students were able to apply the translation concept, some still struggled to formulate the formula symbolically. This highlights the need for conceptual guidance toward abstraction (Akarsu, 2022). Reflective activities and discussions are needed for students to connect concrete experiences with formal concepts (Kandaga et al., 2022), accompanied by strengthening symbolization in HLT (Avcu & Çetinkaya, 2021). Support can be strengthened through inquiry practices that facilitate geometric reasoning in a dynamic learning environment (Uygun, 2020).

It is important to highlight that chess can be a useful tool in education because it involves processes such as visual memory, which can be developed and directed toward concepts of geometric transformation (Sosa & Aguilar, 2021). Through the RME approach, students not only memorize formulas but also develop a deeper understanding of geometric concepts. By using real-world contexts such as chess, teachers can help students see the connection between mathematics and the world around them, making the learning process more meaningful and enjoyable (Adha et al., 2024). Furthermore, geometric reasoning allows students to draw conclusions based on the properties of geometric shapes, and this approach can be used in student learning to help them rediscover a formula (Sahara et al., 2024).

CONCLUSION

Based on the research results, a new finding was obtained in the form of a Hypothetical Learning Trajectory (HLT) to teach the concept of translation using the context of chess. This trajectory consists of four systematic stages, including observing chess game videos and pawn movements as an introduction to the context; representing pawn movements on a Cartesian coordinate plane; identifying the characteristics of translation transformations; and discovering the formulas and concepts of translation. This finding not only provides an overview of the resulting learning stages but also shows that the use of chess contexts can bridge students' understanding from real-life situations to formal stages. Most students successfully constructed their own formulas for translation through the shifting patterns of pawns in the Cartesian coordinate system, reflecting a process of generalization based on geometric reasoning. However, some students experienced difficulties in the process of moving toward symbolism, necessitating the strengthening of reflective activities in HLT. Therefore, the developed HLT not only aids students' conceptual understanding but also functions as a situational learning model that can be applied and empirically tested.

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