Problem-Based and Guided Discovery Approach in Developing a Differential Calculus Textbook

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Abstract

This study was conducted with the primary objective of developing a problem-based differential calculus textbook designed through a guided discovery approach, while simultaneously examining its validity, practicality, and effectiveness. The research employed a research and development (R&D) framework, specifically the 4D model— Define, Design, Develop, and Disseminate—adapted in this study up to the Implementation stage. The participants comprised 20 third-semester students enrolled in the Mathematics Education Program at UIN Syahada Padangsidimpuan. Data collection instruments included validation sheets, response questionnaires, and conceptual understanding tests. Descriptive statistics were applied to evaluate validity and practicality, whereas inferential analyses, including t-tests and N-gain calculations, were performed to assess effectiveness. Findings revealed that the textbook achieved a "highly valid" category with an average expert validation score of 4.5. Practicality testing showed favorable responses from both students and lecturers, with a mean score of 4.4, also classified as "very practical." In terms of effectiveness, the intervention produced a significant improvement in students' learning outcomes: the mean pretest score of 58.2 increased to 82.6 in the posttest, yielding an N- gain value of 0.58, which falls within the medium-to-high category. These results suggest that the developed textbook is feasible, practical, and effective for use in differential calculus instruction. Moreover, the study highlights the potential for broader implementation and recommends further development to cover integral calculus and other mathematical topics.

Keywords: *Differential Calculus; Problem-Based Learning; Guided Discovery, Textbook Development.*

Abstrak

Penelitian ini dilaksanakan dengan tujuan menghasilkan buku ajar kalkulus diferensial berbasis masalah menggunakan pendekatan penemuan terbimbing, sekaligus menilai tingkat validitas, kepraktisan, dan efektivitasnya. Metode yang diterapkan adalah penelitian dan pengembangan (R&D) dengan model 4D (Define, Design, Develop, dan Disseminate), yang dimodifikasi sampai pada tahap Implementasi. Subjek penelitian terdiri atas 20 mahasiswa semester tiga Program Studi Pendidikan Matematika UIN Syahada Padangsidimpuan. Instrumen yang digunakan meliputi lembar validasi, kuesioner tanggapan, dan tes pemahaman konsep. Analisis data dilakukan secara deskriptif untuk menilai validitas serta kepraktisan, dan secara inferensial untuk menguji efektivitas melalui uji-t serta perhitungan N-gain. Hasil penelitian mengindikasikan bahwa buku ajar yang dikembangkan memperoleh penilaian "sangat valid" dengan rata-rata skor 4,5. Uji kepraktisan memperlihatkan respon positif dari mahasiswa maupun dosen, dengan nilai rata-rata 4,4 yang tergolong "sangat

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praktis." Sementara itu, uji efektivitas menunjukkan adanya peningkatan signifikan hasil belajar mahasiswa, ditunjukkan oleh skor rata-rata pretest 58,2 yang naik menjadi 82,6 pada posttest, dengan N-gain 0,58 yang termasuk kategori sedang hingga tinggi. Oleh karena itu, buku ajar ini dinilai layak, praktis, dan efektif untuk digunakan dalam pembelajaran kalkulus diferensial. Penelitian ini juga menyarankan penerapan lebih luas dan pengembangan lanjutan pada materi kalkulus integral maupun topik matematika lainnya.

Kata Kunci: Kalkulus Diferensial; Pembelajaran Berbasis Masalah; Penemuan Terbimbing; Pengembangan Buku Ajar.

INTRODUCTION

Differential calculus stands as a cornerstone within the mathematics education curriculum, providing a solid conceptual framework for comprehending change and dynamical processes in diverse quantitative phenomena. Across disciplines such as science, engineering, economics, and interdisciplinary fields, the notions of derivatives and limits function as indispensable analytical tools to evaluate rates of change, determine extrema, and address optimization problems with precision (Razali et al,.2023; Silitonga and Ramadhani, 2023).

Within the higher education context, mastery of differential calculus has been recognized as a prerequisite for more advanced courses, including integral calculus and multivariable analysis. Recent evidence highlights that insufficient comprehension of differential calculus strongly influences students' achievement in integral calculus, contributing up to 49.1% toward overall academic performance (Wazithah & Nasir, 2025). This empirical finding reinforces the claim that developing competency in differential calculus is not merely of theoretical value but exerts a measurable effect on students' learning outcomes. Furthermore, recent scholarship underscores the need for innovative pedagogical strategies—such as problem-based learning and digital tools—to enhance students' conceptual understanding in calculus, thereby narrowing persistent learning gaps (Wazithah & Nasir, 2025).

University students frequently encounter substantial challenges when dealing with abstract concepts in differential calculus, particularly in interpreting limits and derivatives. Recent findings by Mkhatshwa (2024) revealed that many learners mistakenly perceive the derivative merely as a function rather than a

representation of quantitative change, indicating a persistent cognitive barrier to abstract reasoning . Similarly, traditional teaching practices that prioritize memorization of formulas and procedural routines often reinforce rote learning patterns instead of fostering deep conceptual understanding (Wazithah & Nasir, 2025).

Moreover, the lecture-based mode of instruction, which remains dominant in many mathematics classrooms, has been widely criticized for diminishing students' active engagement. A study by PubMedia (2025) demonstrated that such reliance on teacher-centered delivery discourages learners from participating in discussions, explorations, and problem-solving activities, thus restricting their learning experiences. This overemphasis on passive reception not only reduces motivation but also limits the development of self-regulated learning skills and hampers students' ability to transfer mathematical knowledge into authentic and contextualized situations.

Contemporary instructional materials for differential calculus in higher education are often presented in an abstract manner, with limited contextual relevance for students. Consequently, the content tends to be delivered in a procedural and mechanistic fashion—emphasizing formulas and repetitive examples—without fostering meaningful understanding or the transfer of knowledge to real-world situations (Listiani, 2018). Recent scholarship underscores that such approaches may reduce mathematics to rote symbolic manipulation rather than cultivating conceptual depth (Putra & Supriadi, 2021).

Moreover, most available textbooks and modules provide minimal integration of problem-based learning (PBL) strategies, despite strong evidence that this approach enhances active engagement and problem-solving abilities in mathematics education. The absence of authentic, inquiry-driven tasks narrows the scope of learning to routine exercises—predominantly drill-type problems—that focus on algorithmic skills instead of conceptual exploration.

As a result, students are frequently conditioned to memorize step-by-step procedures rather than develop an understanding of fundamental principles. The lack of guided discovery elements in these materials further limits opportunities for

learners to construct mathematical knowledge independently, thereby hindering the cultivation of critical thinking skills. This pedagogical limitation is particularly problematic within today's paradigm of active and student-centered learning, which prioritizes inquiry, creativity, and the ability to connect mathematical concepts to diverse contexts (Listiani, 2018).

Problem-based learning (PBL) and guided discovery approaches play a pivotal role in enhancing university students' conceptual understanding of differential calculus. Through PBL, learners are actively engaged in solving contextualized problems, which enables them to bridge abstract mathematical concepts with real- life experiences. This process not only sharpens their critical thinking but also strengthens problem-solving abilities (Hmelo-Silver, 2019). In parallel, guided discovery provides a structured pathway for students to uncover mathematical principles under the guidance of instructors or instructional materials, thereby ensuring that learning transcends procedural mastery and fosters a deeper conceptual comprehension (Mayer, 2020). The integration of these two approaches has been shown to promote learner autonomy while simultaneously securing the attainment of instructional goals in an optimal manner (Artigue & Winsløw, 2023; Gorgievski et al., 2020).

Although prior research has underscored the significance of implementing PBL and guided discovery in mathematics education, their application in the development of differential calculus textbooks remains limited (Zhang & Sun, 2022). Evidence from several studies reveals that students benefit substantially from these approaches, particularly in advancing critical thinking and problem-solving competencies (Ismail et al., 2020).

Nonetheless, a review of the literature suggests that the majority of existing calculus textbooks still emphasize conventional methods, often grounded in procedural routines and repetitive exercises (Nugraha et al., 2021). This gap highlights the urgent need to design differential calculus teaching materials that embed PBL and guided discovery strategies in a systematic manner.

The present study therefore aims to design and develop a problem-based differential calculus textbook integrated with guided discovery principles. The

material is not only intended to help students master derivative procedures but also to deepen their conceptual understanding of the underlying mathematical operations. By situating learning in authentic problem contexts and scaffolding students' discovery processes, the textbook aspires to cultivate meaningful comprehension and long-term retention of differential calculus concepts (Hidayati et al., 2021; Panasuk & Leblanc, 2020).

In addition to the development of the textbook, this research also examines its validity, practicality, and effectiveness. Validity is evaluated through expert judgment on content accuracy, linguistic appropriateness, and instructional coherence. Practicality is assessed based on feedback from lecturers and students concerning readability and ease of use during classroom implementation. Finally, effectiveness is measured through learning outcomes, particularly improvements in students' conceptual understanding and problem-solving performance in differential calculus (Razali et al., 2023).

RESEARCH METHODS

This research employed a Research and Development (R&D) design using the 4D model—Define, Design, Develop, and Disseminate—which was further adjusted by incorporating an Implementation phase to align with the study's context. The participants consisted of 20 third-semester students from the Mathematics Education Department (Tadris Matematika) at UIN Syahada Padangsidimpuan. The development process was organized into five essential stages: Analysis, Design, Development, Implementation, and Evaluation.

The instruments utilized in this study included validation sheets, response questionnaires, and a conceptual understanding test. Descriptive statistical methods were applied to evaluate the validity and practicality of the product, while inferential techniques—specifically the t-test and N-gain analysis—were employed to measure its effectiveness (Setyosari, 2020). These analyses provided a comprehensive overview of the product's quality, ensuring that the differential calculus textbook produced met the established standards of being valid, practical, and effective (Yuniarti & Hidayati, 2021).

RESULTS AND DISCUSSION

The expert validation conducted by three evaluators—content, media, and language specialists—indicated that the problem-based differential calculus textbook designed with a guided discovery approach achieved an overall rating within the "highly valid" category. The content dimension was evaluated in terms of alignment with intended learning outcomes, conceptual accuracy, and the comprehensiveness of the topics covered. With respect to the media aspect, the validators emphasized that the book's layout, visual design, and structural organization were sufficiently engaging and systematic, thereby enhancing readability. Meanwhile, the language dimension received feedback highlighting the communicative nature of terminology usage, although certain sections required simplification to ensure accessibility for undergraduate learners.

Following the validation, several revisions were implemented to refine the product. Adjustments included the addition of more representative graphical illustrations for algebraic and trigonometric derivatives, which aimed to strengthen students' ability to connect mathematical concepts with their visual representations. Furthermore, contextual practice problems drawn from science, economics, and everyday life were incorporated into the exercise sections, reinforcing the integration of problem-based learning principles. Minor editorial revisions were also made to achieve greater consistency with academic language standards. These findings corroborate the assertion of Hidayati et al. (2021) that expert validation serves as a critical step to guarantee the appropriateness of content, linguistic features, and media design in instructional material development. More recent scholarship also emphasizes that integrating expert reviews with iterative revisions enhances both the pedagogical value and practical usability of mathematics learning resources (Putra & Alimuddin, 2020).

The practicality of the problem-based differential calculus textbook, developed through a guided discovery approach, was evaluated using response questionnaires distributed to both students and lecturers. Respondents were asked to assess several dimensions, including readability, clarity of material presentation, ease of use, attractiveness, and the perceived benefits of the textbook in supporting

learning activities. The analysis revealed that the overall average score from both groups fell within the "highly practical" category, thereby confirming its feasibility for classroom implementation.

From the students' perspective, the majority indicated that the textbook was easy to comprehend, systematically organized, and effective in stimulating active participation in the learning process. They also emphasized that the inclusion of contextual problems made the material more meaningful and relevant to everyday life. Meanwhile, lecturers highlighted the textbook's flexibility, noting that it could be effectively employed in both face-to-face and online learning environments, while also facilitating the integration of problem-based learning strategies. These findings align with the conclusions of Nugraha et al. (2021), who argued that the practicality of instructional materials is largely determined by their readability, ease of application, and contextual alignment.

Recent studies further support this perspective. For instance, instructional resources that integrate problem-based and contextual tasks have been shown to enhance student engagement and perceived usefulness (Ariyanto et al., 2022). Similarly, the flexibility of textbooks to adapt across learning modes has become increasingly important in the post-pandemic era, where hybrid and online learning continue to shape higher education practices (König et al., 2022). Thus, the developed textbook not only demonstrates high practicality but also reflects contemporary educational demands.

Table 1. Results of Student and Lecturer Responses to the Textbook

Respondents	Mean Score	Category
Students	4.3	Very Practical
Lecturers	4.5	Very Practical
Total	4.4	Very Practical

Note:

Scores are based on a Likert scale of 1–5

1 = not practical

5 = very practical

The effectiveness of the problem-based differential calculus textbook, designed through a guided discovery approach, was evaluated using pretest and posttest scores administered to students. The findings revealed a significant improvement in conceptual understanding after utilizing the textbook. Posttest results demonstrated higher mean scores than the pretest, with normalized gain (N-gain) values ranging from moderate to high. Furthermore, the paired t-test confirmed a statistically significant difference between pretest and posttest results, indicating that the textbook successfully enhanced students' comprehension of derivatives and their applications.

Beyond the quantitative evidence, qualitative observations also suggested an improvement in the overall learning process. Students showed greater participation in classroom discussions, increased confidence in raising questions, and demonstrated more critical engagement when analyzing contextual mathematical problems. These outcomes are consistent with previous research, which highlights the effectiveness of integrating problem-based learning with guided discovery in fostering both conceptual mastery and critical thinking skills among students (Sari & Nusantara, 2022). More recent studies have also emphasized the potential of problem-based and inquiry-driven approaches to stimulate students' active learning behaviors and higher-order thinking skills in mathematics education (Permatasari et al., 2024). Therefore, the textbook can be regarded as an effective instructional tool for enhancing students' learning outcomes and cognitive skills.

Table 2. Results of the Textbook Effectiveness Test

Aspect	Pretest	Posttest	N-gain	Category
Mean Score	58.2	82.6	0.58	Moderate–High
t-test (p < .05)	–	Significant	–	Effective

Note:
N-gain < 0.3 = low
N-gain 0.3–0.7 = moderate;
N-gain > 0.7 = high.

The findings of this study highlight the validity, practicality, and effectiveness of a problem-based differential calculus textbook designed with a guided discovery approach. These three dimensions serve as the primary benchmarks in evaluating the quality of instructional materials, consistent with the framework of research and development (R&D) that emphasizes the necessity of producing learning products that are valid, feasible, and effective.

The validity of the textbook was established through expert reviews conducted by three specialists: a content expert, a media expert, and a language expert. The evaluation results demonstrated an average score categorized as highly valid, with a total rating of 4.5 out of 5. Regarding the content aspect, the material was found to appropriately cover differential calculus concepts aligned with the expected learning outcomes, while effectively integrating relevant contextual problems. From the media perspective, the layout and visual design were judged to be both attractive and facilitative for student engagement in following the learning flow. Linguistically, the text was considered communicative, though minor adjustments were suggested to simplify certain technical terms.

Table 3. Results of Validation Assessment

Aspects Assessed	Mean Score	Category
Content/Material	4.6	Very Valid
Media/Design	4.5	Very Valid
Language	4.4	Very Valid
Overall Average	4.5	Very Valid

The practicality of the textbook was assessed through response questionnaires administered to both students and lecturers. The findings revealed that the material was considered highly practical, with mean scores of 4.3 from students and 4.5 from lecturers. From the learners' perspective, the textbook was regarded as easy to comprehend, logically structured, and effective in fostering active engagement in discussions and problem-solving activities. Conversely, lecturers emphasized that the textbook demonstrated flexibility for use in both face-to-face and online settings, while also being particularly relevant for the

implementation of problem-based learning strategies. These results corroborate previous evidence suggesting that readability, attractiveness, and ease of implementation are among the core indicators of instructional material practicality (Nugraha et al., 2021). Moreover, recent studies further highlight that teaching resources perceived as practical not only enhance learner motivation but also contribute significantly to the effectiveness of innovative pedagogical approaches (Al-Khalifa & AlAjlan, 2023).

Table 4. Results of Student and Lecturer Responses

Respondents	Mean Score	Category
Students	4.3	Very Practical
Lecturers	4.5	Very Practical
Total	4.4	Very Practical

This study produced a problem-based differential calculus textbook designed through the 4D research and development model, incorporating a guided discovery approach. The validation outcomes indicated that the textbook achieved a "highly valid" classification across the dimensions of content, media, and language. Furthermore, the practicality assessment revealed that the material was user- friendly for both students and lecturers, while the effectiveness test demonstrated that it significantly enhanced learners' comprehension of differential calculus concepts. Hence, the textbook can be regarded as valid, practical, and effective as an instructional resource in differential calculus courses.

Beyond its validity, practicality, and effectiveness, the implementation of this textbook has proven to foster students' deeper conceptual understanding of differentiation. Rather than merely achieving procedural mastery, students were able to connect mathematical concepts with real-world problem contexts. Through the integration of problem-based learning and guided discovery, learners were encouraged to engage in critical thinking, exploration, and the development of mathematical problem-solving abilities. This suggests that the textbook functions not only as a medium for knowledge transmission but also as a pedagogical tool that cultivates higher-order thinking skills—an essential competence in

mathematics. Based on these findings, it is recommended that the problem-based, guided discovery differential calculus textbook be more widely adopted in various classrooms and higher education institutions (Tambychik et al., 2020).

Future studies may extend the innovation by developing similar teaching materials for integral calculus or other mathematical domains, thereby broadening the scope of instructional reform. Nonetheless, this research is not without limitations. The relatively small sample size and the restriction to students from a single study program limit the generalizability of the findings. Consequently, further investigations with larger and more diverse cohorts are required to strengthen the external validity of the results (Arifin & Retnawati, 2022)..

CONCLUSION

Based on the research findings, it can be concluded that the developed textbook is valid, practical, and effective for differential calculus instruction. The material achieved a high level of validity with a mean score of 4.5 (highly valid). Practicality assessments revealed positive responses from both students and lecturers, yielding a mean score of 4.4 (highly practical). Furthermore, effectiveness tests demonstrated a significant improvement in student learning outcomes; the mean score rose from 58.2 in the pretest to 82.6 in the posttest, with an N-gain of 0.58 (moderate-to-high category). These results suggest the potential for broader implementation and further development of instructional materials for integral calculus and other mathematical disciplines.

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