

***Factor Analysis of Students' Difficulties
Understanding the Concept of Logarithms
at SMA Negeri 2 Percontohan Karang Baru Aceh Tamiang***

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Abstract

This research aims to determine the factors that cause students' difficulties in understanding the concept of logarithms using the factor analysis method. The samples taken were 120 class X students of SMA N 2 Percontohan Karang Baru Aceh Tamiang. Data was obtained using diagnostic test instruments for logarithmic material based on aspects of logarithms as objects, as processes, as functions, and in contextual problems. From these four aspects, Author 2 was able to identify 20 variables that caused students difficulty in understanding the concept of logarithms. The research results show that there are three factors that cause students' difficulties in understanding the concept of logarithms. These factors are: (a) logarithmic expressions (b) logarithmic functions, and (c) logarithmic calculations. The logarithmic expression factor is the dominant factor which has an eigenvalue of 10.282 which is able to explain a variance value of 54.115%.

Keywords: *Factor Analysis; Students' Difficulties; Concept Understanding; Logarithm.*

Abstrak

Penelitian ini bertujuan untuk mengetahui faktor-faktor yang menyebabkan kesulitan siswa dalam memahami konsep logaritma dengan menggunakan metode analisis faktor. Sampel yang diambil adalah 120 siswa kelas X SMA N 2 Percontohan Karang Baru Aceh Tamiang. Data diperoleh dengan menggunakan instrumen tes diagnostik materi logaritma berdasarkan aspek logaritma sebagai objek, sebagai proses, sebagai fungsi, dan dalam permasalahan kontekstual. Dari keempat aspek tersebut, Penulis 2 mampu mengidentifikasi 20 variabel yang menyebabkan kesulitan siswa dalam memahami konsep logaritma. Hasil penelitian menunjukkan ada tiga faktor yang menyebabkan kesulitan siswa dalam memahami konsep logaritma. Faktor-faktor tersebut adalah : (a) ekspresi logaritma (b) fungsi logaritma, dan (c) perhitungan logaritma. Faktor ekspresi logaritma merupakan faktor dominan yang memiliki nilai eigenvalue sebesar 10,282 yang mampu menjelaskan nilai variance sebesar 54,115%.

Kata Kunci: *Analisis Faktor; Kesulitan Siswa; Pemahaman Konsep; Logaritma.*

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INTRODUCTION

Coverage of mathematics material as a general subject in SMA/SMK includes number systems, algebra, geometry, trigonometry, three-dimensional space, statistics and probability theory, limits, and derivatives, integrals, principles of logic (including mathematical induction), basics of integrals, as well as Mathematical competence in supporting the achievement of high school graduate competence, one of which emphasizes the aspect of "*knowing, exploring and using the properties of operations (including composition) in solving problems of systems of equations and inequalities, power and logarithmic functions, algebraic functions, assisted by geometric techniques and interpretations*" (Pujiadi, 2016).

Logarithm which is an aspect of algebra is no exception to the benefits algebra offers (Ansah & Vos, 2016). It finds application in areas such as engineering (Aaron, 1997), medicine (Kinsella, 2007), economics (Azad, 2012), etc.

Despite the benefits associated with logarithms, according to (Wong, Lam, Wong, Leung, & Mok, 2002) most students think logarithms are something that is difficult and has no use in everyday life. (Liang & Wood, 2005) also stated that the topic of logarithms is a difficult topic for students and there is a high prevalence of misconceptions in students' thinking.

The difficulty can lie in one of the four mathematical objects, namely concepts. A concept is an abstract idea that allows someone to classify an object and explain whether the object is an example or not an example of an abstract idea. (Yen, 1999) stated that students often see the notation "*log*" as an object, not an operation. (Hurwitz, 1999) notes that students often have difficulty thinking of logarithms as the output of a function because the notation used for logarithms does not look like the usual $f(x)$ notation. (Gamble, 2005) said that teachers often tell students that "logarithms are exponents", but for some reason, students hear the terms "exponent" and "logarithm" but often do not understand the relationship between the two. (Berezovski, 2006) found that students' difficulties in learning logarithms were caused by students focusing more on the procedural approach and relying too much on rules rather than the concept of logarithms themselves.

Then (Berezovski & Zazkis, 2006) found that students had difficulty recognizing logarithms as numbers. They expressed doubt that problems in calculating logarithmic expressions involving only numbers using either a calculator or manual calculations interpret the understanding of logarithms as numbers. Students may have learned the procedure by which certain types of problems are presented but do not understand that logarithmic expressions are numbers.

This is in line with research by (Kastberg, 2002) which reports that students who can solve problems do not necessarily perceive logarithmic expressions as numbers. (Tabaghi, 2007), in his research identifying students' difficulties in understanding the concept of logarithms using the context of APOS theory, found that most students' understanding of the meaning of logarithmic arithmetic did not go beyond the "process" level, because understanding the meaning of logarithmic arithmetic as exponential cancellation requires an understanding of exponentials with real exponents. (Hoon, Singh, & Ayop, 2010) in their research found that students were able to do routine calculations in logarithms, but were less able to solve problems that required high-level cognitive thinking. (Ganesan & Dindyal, 2014) stated that students' errors in logarithms can be related to their understanding of the topic which implies knowledge of their mathematical thinking and mental constructs. (Weber, 2016) focuses on logarithms as numbers or operators, where logarithms can be conceptualized as inverse exponents (*i.e.* structurally). Since the indirect definition is still valid in algebra textbooks despite the many problems reported in the educational literature, it may be a major cause of student difficulties. (Dintarini, 2018) found that some students did not fully understand the meaning of logarithms. In the end, students had difficulty applying the definition of logarithms to problems. At first, students seemed able to calculate the logarithm value of a number but failed when faced with more complex logarithms.

Meanwhile, (Rafi & Retnawati, 2018) found that the common mistakes students made in logarithmic material were technical errors, distorted theorems or definitions, unverified solutions, and data usage errors. (Liang & Wood, 2005) also found that students appeared to be able to perform routine calculations but

were less able to answer questions that required a higher level of cognitive thinking. In addition, many errors are not caused by a lack of knowledge but appear to be due to an over-generalization of algebraic rules. Students tend to have low understanding regarding the initial concept of logarithmic functions, students have difficulty understanding the application of logarithmic functions, and difficulties in solving logarithmic functions.

According to (Hayati & Budiyo, 2018), the possible causes of errors in logarithmic learning come from teachers, students, or learning tools. Student errors in completing several tasks related to logarithmic function material became an obstacle in the next learning process.

According to (Sinaga, et al., 2014) students must be able to master the concepts and properties of logarithms before studying logarithmic functions. Then, what are the dominant factors that actually cause students to have difficulty understanding the concept of logarithms? This question is what makes the author interested in exploring students' difficulties in understanding logarithmic material at SMA Negeri 2 Percontohan Karang Baru Aceh Tamiang.

RESEARCH METHODS

The location of this research was conducted at the SMA Negeri 2 Percontohan Karang Baru Aceh Tamiang. 120 class X students were taken as a sample. To obtain research data, Author 2, who is a Mathematics teacher at the research location, used a diagnostic test instrument for logarithmic material based on the aspects proposed by (Williams, 2011), namely the logarithm is divided into four parts; logarithms as objects, as processes, as functions, and in contextual matters. Furthermore, from these four aspects, Author 2 compiled a questionnaire and succeeded in identifying 20 factor items that caused class X students to have difficulty understanding the concept of logarithm. The indicators can be seen in Table 1:

Table 1. Indicators of Students' Difficulties in Understanding the Concepts of Logarithm

Index	Indicator
X ₁	know the meaning of ${}^a\log b$
X ₂	know that logarithmic expressions are numbers, and don't need to be approximated by decimals
X ₃	flexibly change the form of logarithmic expressions using logarithm rules
X ₄	know the rules of logarithmic notation and how they relate to the order of operations
X ₅	knows that a logarithmic expression must have a base and an argument
X ₆	simplifies logarithmic expressions
X ₇	estimate the value of a logarithmic expression
X ₈	use logarithms by eliminating the base of exponential expressions when solving exponential equations
X ₉	uses logarithms to find the value of the exponent in an expression
X ₁₀	Use logarithms in the repeated division problem
X ₁₁	connecting square (or other) roots with logarithms
X ₁₂	"insert" a value in the function domain that will produce a single value
X ₁₃	know logarithm function graphs in general and understand how to make logarithm graphs
X ₁₄	understand that logarithmic functions have a finite domain, and an infinite range
X ₁₅	explains why there are vertical asymptotes, but no horizontal asymptotes, in logarithmic graphs
X ₁₆	connect logarithmic functions and exponential functions as inverses
X ₁₇	understand the constraints of solutions to equations and inequalities involving logarithms
X ₁₈	solve real-world problems involving logarithmic properties
X ₁₉	recognize when real-world problems are most easily solved using logarithms
X ₂₀	explains why logarithms are useful in the real world

A quantitative approach is used to analyze the collected data so that the factors that cause difficulties in understanding the logarithmic concept can be

identified. The *Explanatory Factor Analysis* (EFA) type factor analysis method was chosen to reduce the variables represented by the questionnaire constituent items and the *Principal Components Analysis* (PCA) technique was used to form the factors. According to (Johnson & Wichern, 2007), *Principal Components Analysis* (PCA) is the simplest procedure for extracting factors. This method forms a linear combination of observed indicators.

RESULTS AND DISCUSSION

Based on (Sugiyono, 2008), a variable is said to be valid for use as a reflection of a concept if its correlation value with the total other variables is ≥ 0.30 and the concept being measured can be trusted if the value of Cronbach's Alpha is ≥ 0.60 . The validity of the items and the reliability of the questionnaire were tested with the SPSS 20 application. The results of the validity and reliability tests are shown in Table 2.

In Table 2, it can be seen that of all the variables, there is one variable that is invalid, namely variable if variable X_5 is removed, it will change the *Cronbach's Alpha* value from 0.949 to 0.951 so that variable X_5 can be removed. Taking into account the 19 remaining questionnaire statement items after the X_5 indicator was issued, the reduction of the 19 items into the factors they represent is carried out by *Explanatory Factor Analysis* (EFA) through the stages as explained by (Hair, Black, Babin, & Anderson, 2010).

The data feasibility test is used to see whether the data in the questionnaire is suitable for processing using factor analysis. The data suitability test is determined by looking at the Bartlett test of sphericity, *Kaiser Meyer-Olkin* (KMO), and *Measures of Sampling Adequacy* (MSA) values that are sought for each indicator. The quantity values that must be met according to (Johnson & Wichern, 2007) the *Bartlett test of sphericity* value with significance < 0.05 , the KMO value must be ≥ 0.50 and the MSA value must be ≥ 0.50 . The KMO value obtained from all variables is 0.913, where the KMO value meets the requirements. In general, factor analysis can be used on all variables formed. The significance value in the Bartlett test is $0.000 < 0.05$, so it is concluded that the

correlation between the variables that form factors can be explained by other variables.

Table 2. Validity and Reliability Test

Index	Variable	Item Total Correlation	Cronbach's Alpha if < 0.60, Item deleted
X ₁	know the meaning of ${}^a\log b$	0.749	0.945
X ₂	know that logarithmic expressions are numbers, and don't need to be approximated by decimals	0.805	0.944
X ₃	flexibly change the form of logarithmic expressions using logarithm rules	0.823	0.944
X ₄	know the rules of logarithmic notation and how they relate to the order of operations	0.707	0.946
X ₅	knows that a logarithmic expression must have a base and an argument	0.142	0.951
X ₆	simplifies logarithmic expressions	0.728	0.945
X ₇	estimate the value of a logarithmic expression	0.394	0.95
X ₈	use logarithms by eliminating the base of exponential expressions when solving exponential equations	0.696	0.946
X ₉	uses logarithms to find the value of the exponent in an expression	0.494	0.949
X ₁₀	Use logarithms in the repeated division problem	0.808	0.944
X ₁₁	connecting square (or other) roots with logarithms	0.790	0.944
X ₁₂	"insert" a value in the function domain that will produce a single value	0.627	0.947
X ₁₃	know logarithm function graphs in general and understand how to make logarithm graphs	0.861	0.943
X ₁₄	understand that logarithmic functions have a finite domain, and an infinite range	0.645	0.947

Index	Variable	Item Total Correlation	Cronbach's Alpha if < 0.60, Item deleted
X ₁₅	explains why there are vertical asymptotes, but no horizontal asymptotes, in logarithmic graphs	0.854	0.943
X ₁₆	connect logarithmic functions and exponential functions as inverses	0.409	0.95
X ₁₇	understand the constraints of solutions to equations and inequalities involving logarithms	0.622	0.947
X ₁₈	solve real-world problems involving logarithmic properties	0.740	0.945
X ₁₉	recognize when real-world problems are most easily solved using logarithms	0.472	0.949
X ₂₀	explains why logarithms are useful in the real world	0.849	0.943

The next step is to carry out the MSA test, according to (Johnson & Wichern, 2007) the MSA value must be ≥ 0.50 , in this study the MSA criteria used was > 0.60 . The MSA test for all variables can be seen in Table 3.

The results of the MSA test state that each indicator has an MSA value, which means that each indicator meets the MSA requirements, so that the factor analysis process can be continued.

To determine the number of factors formed, factors with an eigenvalue of more than 1 are the factors that will be selected. The greater the eigenvalue of a factor, the better the factor is at representing a number of variables. By using the *Principal Component Analysis* (PCA) method, factors with an eigenvalue of more than 1 can be seen in Table 4.

In Table 4 it can be seen that there are 3 factors with a cumulative percentage of 67.338% which are able to explain the factors that cause difficulties in understanding the concept of logarithms.

Table 3. MSA Value of All Indicators

Index	Variable	MSA Value
X ₁	know the meaning of ${}^a\log b$	0.887
X ₂	know that logarithmic expressions are numbers, and don't need to be approximated by decimals	0.966
X ₃	flexibly change the form of logarithmic expressions using logarithm rules	0.918
X ₄	know the rules of logarithmic notation and how they relate to the order of operations	0.883
X ₆	simplifies logarithmic expressions	0.838
X ₇	estimate the value of a logarithmic expression	0.820
X ₈	use logarithms by eliminating the base of exponential expressions when solving exponential equations	0.938
X ₉	uses logarithms to find the value of the exponent in an expression	0.893
X ₁₀	Use logarithms in the repeated division problem	0.955
X ₁₁	connecting square (or other) roots with logarithms	0.968
X ₁₂	"insert" a value in the function domain that will produce a single value	0.953
X ₁₃	know logarithm function graphs in general and understand how to make logarithm graphs	0.941
X ₁₄	understand that logarithmic functions have a finite domain, and an infinite range	0.868
X ₁₅	explains why there are vertical asymptotes, but no horizontal asymptotes, in logarithmic graphs	0.955
X ₁₆	connect logarithmic functions and exponential functions as inverses	0.741
X ₁₇	understand the constraints of solutions to equations and inequalities involving logarithms	0.935
X ₁₈	solve real-world problems involving logarithmic properties	0.843
X ₁₉	recognize when real-world problems are most easily solved using logarithms	0.909
X ₂₀	explains why logarithms are useful in the real world	0.950

Table 4. Number of Factors Formed

Component	Initial Eigenvalues		
	Total	% of Variance	Cumulative %
1	10.282	54.115	54.115
2	1.363	7.175	61.290
3	1.149	6.048	67.338

In this study, factor analysis with *Principal Component Analysis* (PCA) extraction obtained 3 factors that cause difficulties in understanding the concept of logarithms. These factors have a total of 19 variables. The three factors in the *Principal Component Analysis* (PCA) extraction method are each given a name according to the grouping of results on factor rotation. The results of the *varimax factor rotation* can be seen in Table 5.

Table 5. Results of Factor Rotation with the Varimax Method

Factor 1		Factor 2		Factor 3	
Variable	Loading Value	Variable	Loading Value	Variable	Loading Value
X ₆ (simplifies logarithmic expressions)	0.840	X ₁₆ (connect logarithmic functions and exponential functions as inverses)	0.858	X ₇ (estimate the value of a logarithmic expression)	0.778
X ₁₈ (solve real-world problems involving logarithmic properties)	0.827	X ₁₄ (understand that logarithmic functions have a finite domain, and an infinite range)	0.735	X ₉ (uses logarithms to find the value of the exponent in an expression)	0.728
X ₁ (know the meaning of ^a log b)	0.812	X ₁₉ (recognize when real-	0.588	X ₁₂ ("insert" a value in the	0.488

Factor 1		Factor 2		Factor 3	
Variable	Loading Value	Variable	Loading Value	Variable	Loading Value
		world problems are most easily solved using logarithms)		function domain that will produce a single value)	
X ₂₀ (explains why logarithms are useful in the real world)	0.799	-		-	
X ₄ (know the rules of logarithmic notation and how they relate to the order of operations)	0.770	-		-	
X ₂ (know that logarithmic expressions are numbers, and don't need to be approximated by decimals)	0.724	-		-	
X ₁₃ (know logarithm function graphs in general and understand how to make logarithm graphs)	0.707	-		-	
X ₁₅ (explains why there are vertical asymptotes, but no horizontal	0.689	-		-	

Factor 1		Factor 2		Factor 3	
Variable	Loading Value	Variable	Loading Value	Variable	Loading Value
asymptotes, in logarithmic graphs)					
X ₃ (flexibly change the form of logarithmic expressions using logarithm rules)	0.672	-		-	
X ₁₀ (use logarithms in the repeated division problem)	0.653	-		-	
X ₁₁ (connecting square (or other) roots with logarithms)	0.630	-		-	
X ₁₇ (understand the constraints of solutions to equations and inequalities involving logarithms)	0.609	-		-	
X ₈ (use logarithms by eliminating the base of exponential expressions when solving exponential equations)	0.597	-		-	

In variable X_{12} (entering a value in the function domain which will produce a single value) it is shown that the loading value is 0.488, based on the provisions of the loading value according to (Hair, Black, Babin, & Anderson, 2010) it is explained that for a sample of 120 students the recommended loading value is 0.50, so that the variable X_{12} cannot be included in the factor because the loading value is below 0.50.

Next, factor interpretation is carried out by grouping indicators that have a minimum weighting factor of 0.50, and the variables that have a weighting factor of less than 0.50 will be removed from the model. Based on Table 4, it can be seen that there are 18 variables spread across 3 factors with a total variance of 67.338%. Based on the results of the *varimax rotation*, it can be seen that the variables in each factor that cause difficulties in understanding the concept of logarithms form three factors. Each of the three factors is given a new name according to the grouping of variables in that factor. However, it should be noted that the factor naming process is not very scientific and is based on the subjective opinion of the analyst (Hair, Black, Babin, & Anderson, 2010).

Factor 1: The first factor is called *the logarithmic expression* factor which has an eigenvalue of 10.282 and can explain a variance value of 54.115%. This means that 54.115% of the difficulties in understanding the concept of logarithms are determined by *the logarithmic expression* factors. This factor is formed from 13 variables based on Table 5.

Factor 2: Indicator the second factor is called *the logarithmic function* which has an eigenvalue of 1.363 and is able to explain a variance value of 7.175%. This means that 7.175% of the difficulties in understanding the concept of logarithms are determined by *the logarithmic function* factors. This factor is formed from three indicators, with the X_{16} indicator (*connecting the logarithm function and exponential function as an inverse*) having the largest loading factor value with a value of 0.858. It can also be interpreted that the difficulty of understanding the relationship between logarithmic functions and exponential functions will make it increasingly difficult for students to solve logarithms in the real world.

Factor 3: The third factor is called *the logarithmic calculation* which has an eigenvalue of *1.149* and can explain a variance value of *6.048%*. This means that *6.048%* of the difficulties in understanding the concept of logarithms are determined by *the logarithmic calculation factors*. This factor is formed from 2 indicators with indicator *X₇ (estimating the logarithmic expression value)* which has the largest loading factor value with a value of *0.778*. This means that to be able to estimate the value of a logarithmic expression, students must have calculation skills in mathematics lessons.

CONCLUSION

Research on the factors that cause difficulty understanding the concept of logarithms in class X students of SMA N 2 Percontohan Karang Baru Aceh Tamiang concluded that; there are three factors identified that cause difficulties in understanding the concept of logarithms. The three factors extracted from the 19 original variables are: (a) *logarithmic expression*, (b) *logarithmic function*, and (c) *logarithmic calculation*. The logarithmic expression factor is the dominant factor which has an eigenvalue of *10,282* which can explain the variance value of *54,115%*.

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